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MODAL CONTROL
AND
EIGENVALUE ASSIGNMENT

by

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A THESIS

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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research, for acceptance, a thesis entitled MODAL CONTROL AND EIGENVALUE ASSIGNMENT submitted by TOROS TOPALOGLU in partial fulfilment of the requirements for the degree of Master of Science.

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ABSTRACT

This thesis presents a new algorithm for assigning closed-loop eigenvalues of multivariable linear systems using output feedback control. Subject to certain mild restrictions, the number of eigenvalues which can be assigned to arbitrary, distinct locations is min [m+r-1,n] where m, r, and n are the dimensions of the output, control and state vectors, respectively. The algorithm provides an extension of previous results but allows a significantly large number of eigenvalues to be assigned.

An extensive literature survey of modal control and eigenvalue assignment methods based on proportional feedback control is included. In this survey the existing methods are classified, described, interpreted and critically evaluated. Furthermore, the design options and design parameters in each method are identified and their usefulness discussed.

Digital simulation studies involving the application of representative modal control methods and the new eigenvalue assignment method to the control of a double effect pilot plant evaporator model are described. In these studies the effects of the various design options on the response characteristics are demonstrated.

Finally, concluding comments on the proper use of the design options are presented and fundamental differences between modal control and eigenvalue assignment are discussed.

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CHAPTER ONE

INTRODUCTION

Significant theoretical advances have been made in recent years in the control of linear multivariable systems. However, few of the methods developed have been of much practical value in the unique control problems facing the process control engineer. First of all, the physichochemical characteristics of the processes are not well understood and thus their modeling is extremely difficult. Even if a reasonably accurate state space representation of the system can be realized and thus application of modern control theory made possible, the resulting model often contains a large number of interacting state variables of which only a limited number can be accurately measured. But, for the application of most of the modern control methods, measurement of all of the state variables is a prerequisite. In the relatively few multivariable control methods where only a subset of the state variables has to be measured, the control configuration to be used is not specified, i.e., the design method does not provide any guidance concerning which variables to measure and which ones to manipulate. These are not the only difficulties facing the process control engineer, but they are the more pronounced ones.

Modal control and eigenvalue assignment methods have been greatly welcomed by process control engineers since they promise to cope with the above problems in those cases where the state space model of the process is available.

The major objective of modal control and eigenvalue assignment methods is to increase the degree of stability and the speed of response of the process. Modal control also provides useful guidance concerning the configuration of the control system which will achieve these objectives.

Recently, objections have been raised against modal control by researchers who have developed the multivariable counterparts of the classical frequency domain control theory [1]. It has been claimed that modal control and eigenvalue assignment methods cannot deal with certain other control problems arising in process control. This is certainly true. In fact, no multivariable control method is capable of achieving all of the desired objectives in a process control design problem.

It is the author's opinion that modal control theory and eigenvalue assignment methods possess certain design options which can be purposefully used to achieve control objectives in addition to those of increasing the degree of stability and the speed of response of the controlled processes. Further developments in this area will certainly add to the versatility of modal control and eigenvalue assignment methods.

1.1 Objectives of the Study

This study is primarily concerned with modal control and eigenvalue assignment methods which employ proportional feedback only. Special emphasis is given to those methods which are applicable to systems with a restricted set of measurements and controls.

The large number of publications available in this area has made the correlation of the available results rather difficult. In fact, even the distinction between modal control and eigenvalue assignment has been largely overlooked by many. It has been one of the intentions of this study to classify, describe, interpret and critically evaluate the available methods in some detail, and thus prepare a basis for future work in this area.

By combining some ideas inherent in modal control and eigenvalue assignment,the author has been able to devise an algorithm, which, subject to certain mild restrictions, significantly increases the number of eigenvalues assignable to arbitrary, distinct locations. It is hoped that this extension to the available methods will prove helpful, especially, in its application to systems of larger dimensionality. It is also hoped that the basic idea employed in this algorithm will be useful in other applications.

Workers in different fields of modern control theory have criticized modal control and eigenvalue assignment methods for providing, in general, a nonunique controller matrix for the same set of closed-loop eigenvalues. It is one of the objectives of the simulation studies in this thesis to demonstrate that this lack of uniqueness may in fact be very useful in fulfilling other design objectives in addition to eigenvalue assignment.

Another aim of this thesis is the application of some of the modal control and eigenvalue assignement techniques to the pilot plant evaporator system in the Department of Chemical Engineering of the University of Alberta to evaluate these techniques in this particular application. Although extensive simulation studies could be performed, the experimental evaluation of the results had to be postponed because of the equipment difficulties.

1.2 Structure of the Thesis

The thesis has been organized in the same sequence that was used in the actual investigation.

In Chapter Two, the existing methods for modal control and eigenvalue assignment have been surveyed, compared and critically evaluated. In Chapter Three, a new eigenvalue assignment algorithm has been presented. In Chapter Four, the results of a simulation case study involving the methods of Chapter Two and Chapter Three are given. Finally, in Chapter Five, the overall conclusions of the thesis have been summarized.

1.3 Preliminary Comments

In this thesis no attempt has been made to dwell on the theoretical concepts of control theory which have been extensively used in modal control and eigenvalue assignment methods. These can be found in standard texts on modern control theory, such as [2].

The methods described in this thesis have been based on the state space representation of linear, lumped-parameter, time invariant continuous-time systems. Their application in the simulation studies of Chapter Four has involved though discrete-time models derived from their continuous-time counterparts. The justification of this approach can be found in [3, 4].

CHAPTER TWO

MODAL CONTROL AND EIGENVALUE ASSIGNMENT: LITERATURE SURVEY AND THEORY

2.1 Introduction

Modal control has been suggested by Rosenbrock [1] as a design technique to cope with some of the special control problems involved in process control. It aims at improving the stability and speed of response of interacting processes. Modal control is not used to eliminate the interactions involved in a multivariable system, instead it aims at controlling directly the modes of the system, and provides information about the control configuration that should be used for the achievement of its aims.

Eigenvalue assignment techniques have been suggested as an approach to fulfill the same objectives. Both methods are capable of achieving the same objectives in cases where the designer has access to the values of all the state variables of the system. In all other cases the two methods are not expected to be equally helpful to the designer. In fact a combination of the two approaches has been successfully realized in the control literature.

It is the intention of this chapter to classify, describe, interpret, and critically evaluate these methods in some detail. The author hopes to identify the available design freedoms and design parameters in each method. The criticisms and appraisals are subjective and the pertinent references have been cited to help the reader to form his own opinions.

2.2 The Modes of a Multivariable System

Consider the following state space representation of a linear time-invariant, lumped-parameter multivariable dynamic system,

$$\underline{\dot{x}}(t) = \underline{\underline{A}}\,\underline{x}(t) + \underline{\underline{B}}\,\underline{u}(t) + \underline{\underline{D}}\,\underline{d}(t) \tag{2.1}$$

$$\underline{y}(t) = \underline{\underline{C}}\,\underline{x}(t) \tag{2.2}$$

where $\underline{x}(t)$ is the $n \times 1$ dimensional state vector,

$\underline{u}(t)$ is the $r \times 1$ dimensional control vector,

$\underline{y}(t)$ is the $m \times 1$ dimensional output vector,

and $\underline{d}(t)$ is the $p \times 1$ dimensional disturbance vector.

The state variables can be viewed as components of the state vector defined relative to the Euclidean basis of the state space. In general, this particular basis results in interacting state variables, i.e., the response of a state variable to disturbance inputs and non-zero initial conditions depends on the response of the other state variables.

Consideration of the solution to (2.1) namely,

$$\underline{x}(t) = e^{\underline{\underline{A}}t}\underline{x}(0) + \int_0^t e^{\underline{\underline{A}}(t-\tau)}\,\underline{\underline{B}}\,\underline{u}(\tau)d\tau + \int_0^t e^{\underline{\underline{A}}(t-\tau)}\underline{\underline{D}}\,\underline{d}(t)d\tau \tag{2.3}$$

reveals another well-known fact: Not only does the excursion of a controlled state variable from its steady state value introduce deviations in other controlled state variables, but also the control action taken to encounter this first excursion directly affects the other controlled state variables, as well.

One possible approach to these problems, modal control, has been suggested by Rosenbrock [1]. It is based on the idea of finding another basis for the state space with the favorable property that the components of the state vector with respect to this new basis are noninteracting. In other words, the new problem consists of the determination of n linear combinations of the original state variables which are decoupled from each other.

This decoupling basis is provided by the n linearly independent right eigenvectors, $\underline{w}_1, \underline{w}_2, \ldots, \underline{w}_n$, of $\underline{\underline{A}}$ [1,2], some of which may be generalized right eigenvectors for the case of repeated eigenvalues [3]. The new noninteracting variables, $z_1, z_2, \ldots, z_n$, are called the canonical or modal state variables and are given by the linear transformation

$$\underline{x} = \underline{\underline{W}}\,\underline{z} \tag{2.4}$$

or

$$\underline{z} = \underline{\underline{W}}^{-1}\,\underline{x}\ . \tag{2.5}$$

Recognition of the fact that the right eigenvectors and left eigenvectors of $\underline{\underline{A}}$, namely, $\underline{v}_1, \underline{v}_2, \ldots, \underline{v}_n$, if normalized, form a biorthonormal set [2], i.e.,

$$\langle \underline{v}_i, \underline{w}_j \rangle = \delta_{ij} \quad (i,j = 1,2,\ldots,n) \tag{2.6}$$

allows (2.5) to be written as

$$\underline{z} = \underline{\underline{V}}^T\,\underline{x}\ . \tag{2.7}$$

Application of this linear transformation to (2.1) and (2.2) yields:

$$\dot{\underline{z}}(t) = \underline{\underline{V}}^T \underline{\underline{A}}\,\underline{\underline{W}}\,\underline{z}(t) + \underline{\underline{V}}^T \underline{\underline{B}}\,\underline{u}(t) + \underline{\underline{V}}^T \underline{\underline{D}}\,\underline{d}(t) \tag{2.8}$$

$$\underline{y}(t) = \underline{\underline{C}}\,\underline{\underline{W}}\,\underline{z}(t) \tag{2.9}$$

or, if $\underline{\underline{A}}$ possesses distinct eigenvalues,

$$\dot{\underline{z}}(t) = \underline{\underline{\Lambda}}\ \underline{z}(t) + \underline{\underline{H}}\ \underline{u}(t) + \underline{\underline{E}}\ \underline{d}(t) \tag{2.10}$$

$$\underline{y} = \underline{\underline{F}}\ \underline{z}(t) \tag{2.11}$$

where

$$\underline{\underline{\Lambda}} = \underline{\underline{V}}^T\ \underline{\underline{A}}\ \underline{\underline{W}} \tag{2.12}$$

$$\underline{\underline{H}} = \underline{\underline{V}}^T\ \underline{\underline{B}} \tag{2.13}$$

$$\underline{\underline{E}} = \underline{\underline{V}}^T\ \underline{\underline{D}} \tag{2.14}$$

and

$$\underline{\underline{F}} = \underline{\underline{C}}\ \underline{\underline{W}} \tag{2.15}$$

$\underline{\underline{\Lambda}}$ is a diagonal matrix, whose diagonal elements consist of the eigenvalues of the system matrix, $\underline{\underline{A}}$. If $\underline{\underline{A}}$ possesses repeated eigenvalues, then the transformation of (2.12) will either result in a diagonal or Jordan canonical matrix. The latter case, which involves analytical difficulties, may be considered as a limiting case as suggested by Rosenbrock [1].

The important outcome of the modal transformation can be depicted by rewriting (2.8) as:

$$\dot{\underline{z}}(t) = \underline{\underline{\Lambda}}\ \underline{z}(t) + \underline{s}(t) + \underline{t}(t) \tag{2.16}$$

where

$$\underline{s}(t) = \underline{\underline{H}}\ \underline{u}(t) \tag{2.17}$$

and

$$\underline{t}(t) = \underline{\underline{E}}\ \underline{d}(t)\ . \tag{2.18}$$

Thus, the transformation of the control vector, $\underline{u}(t)$, the disturbance vector, $\underline{d}(t)$, and the output vector, $\underline{y}(t)$, into the modal inputs, $\underline{s}(t)$ and $\underline{t}(t)$, and the modal state vector, $\underline{z}(t)$, has created n decoupled first order systems,

$$\dot{z}_i(t) = \lambda_i z_i + s_i + t_i, \quad (i = 1,2,\ldots,n) \tag{2.19}$$

from a complex system with interacting inputs and outputs [4-6].

The application of the transformations given in (2.4)-(2.7) and (2.12) to (2.3) leads to the representation of the system response in terms of its modal components [4]:

$$\underline{x}(t) = \sum_{i=1}^{n} [\underline{v}_i^T \underline{x}(0) + \int_0^t \underline{v}_i^T \underline{\underline{B}}\, \underline{u}(\tau) e^{-\lambda_i \tau} d\tau + \int_0^t \underline{v}_i^T \underline{\underline{D}}\, \underline{d}(\tau) e^{-\lambda_i \tau} d\tau] e^{\lambda_i t} \underline{w}_i \,. \tag{2.20}$$

Thus, the system response can be represented as a linear combination of the terms $e^{\lambda_i t} \underline{w}_i$, which will be called the modes of the system, i.e.,

$$\underline{x}(t) = \sum_{i=1}^{n} \xi_i(t) e^{\lambda_i t} \underline{w}_i = \sum_{i=1}^{n} \xi_i(t) \underline{m}_i(t) \tag{2.21}$$

where $\underline{m}_i(t) \equiv e^{\lambda_i t} \underline{w}_i$ is the i^{th} mode, and $\xi_i(t)$ is defined to be the term in brackets in (2.20).

This definition of the mode is not unique. Many authors have defined the modes of the system to be the eigenvalues or the eigenvectors of the system matrix, $\underline{\underline{A}}$. The definition used here is due to Zadeh and Desoer [2], and has been found to simplify some of the following discussions.

When only step changes in both the disturbance and control inputs are considered (2.20) can be simplified to [4]:

$$\underline{x}(t) = [\sum_{i=1}^{n} \underline{v}_i^T \underline{x}(0) e^{\lambda_i t} - (\frac{\underline{v}_i^T \underline{\underline{B}}\, \underline{u}}{\lambda_i} + \frac{\underline{v}_i^T \underline{\underline{D}}\, \underline{d}}{\lambda_i})(1-e^{\lambda_i t})]\underline{w}_i \tag{2.22}$$

which is possibly the most commonly encountered case in practice.

Furthermore, if some of the modes are complex, these modes contribute to the response as though they were real modes equal to the sum and the difference of the two complex conjugate modes. They also contribute sinusoidal oscillations at the frequency determined by the imaginary part of the complex eigenvalue [4].

The magnitude of the contribution made by each mode to the state vector is called the modal activation [4,6], and it consists of:

i) activation of the i^{th} mode due to nonzero initial states, $\underline{v}_i^T \underline{x}(0)$.

ii) activation of the i^{th} mode due to disturbance inputs, $\int_0^t \underline{v}_i^T \underline{\underline{D}}\, \underline{d}(\tau) e^{-\lambda_i \tau} d\tau$.

iii) activation of the i^{th} mode due to control action $\int_0^t \underline{v}_i^T \underline{\underline{B}}\, \underline{u}(\tau) e^{-\lambda_i \tau} d\tau$.

Furthermore, comparison of (2.4) and (2.21) suggests that

$$\underline{x}(t) = \sum_{i=1}^{n} z_i(t)\, \underline{w}_i \quad . \tag{2.23}$$

The above considerations lead to some important conclusions for systems with distinct eigenvalues:

1) The modes respond to initial disturbances and process inputs independently of each other, i.e., to determine the modal activation of the i^{th} mode one makes use of the corresponding left eigenvector $\underline{v}_i$ only.

2) Each mode is associated with a unique time constant, which is the negative reciprocal of the real part of the eigenvalue; for a system to be asymptotically stable all of its eigenvalues must have negative real parts.

3) The free response of each mode for a stable system with real and distinct eigenvalues follows a simple exponential decay, but the free response of the state variables may show a more complicated behavior. This is due to the fact that state variables may have positive components in one mode, and negative components in another.

2.3 Ideal Modal Control

Since the state vector is a linear combination of the right eigenvectors (cf., (2.23)), one way of driving it towards the origin of the state space (the system steady state) is by driving the time-varying coefficients in this linear combination, namely the modal state variables towards zero, i.e., by counteracting the modal activations. This is the basic idea behind modal control.

Thus modal control in its ideal form has to perform the following three tasks [6]:

1) Determination of the value of the modal state, or equivalently, the modal activations, from the system outputs (analysis of the outputs).

2) Application of a linear feedback control law to the modal state vector which results in a modal control vector, $\underline{s}$. This proportional linkage between activation and manipulation is chosen so that the control action is zero when the modal activation is zero. Thus modal control as treated here belongs to the class of proportional controllers.

3) Sythesis of a control vector, $\underline{u}$, from the modal control vector, $\underline{s}$, such that each element of $\underline{u}$ acts on one and only one mode (synthesis of the controls).

Two comments are worth mentioning at this point:

i) Since ideal modal control is a strictly proportional-type control it will invariably cause steady-state offsets in the controlled variables, $\underline{x}$.

ii) Driving the modal state variables towards the origin at all times does not necessarily imply that the actual state variables will be driven towards their steady-state values at all times. Although the state variables will finally reach their steady state values, there may be significant excursions from the origin during this process (the "peaking effect").

The action of the ideal modal controller on ℓ of the system modes can be described by the block diagram in Figure 2.1.

The $\underline{\underline{B}}$ and $\underline{\underline{C}}$ matrices in Figure 2.1 are part of the process and thus fixed, while the controller matrices $\underline{\underline{P}}$, $\underline{\underline{K}}$, and $\underline{\underline{R}}$ may be chosen in any desired way, except that $\underline{\underline{K}}$ must be a diagonal matrix.

Suppose that there are n measurements (i.e., $m = n$) and n controls (i.e., $r = n$), and that the process is both controllable and observable. It is under these circumstances that all modes are independently available for observation and manipulation. The equivalent condition which makes this possible is that both the $\underline{\underline{B}}$ and $\underline{\underline{C}}$ matrices be of rank n.

Task 1 can only be fulfilled if $\underline{p}_i^T$ satisfies

$$\underline{p}_i^T \underline{\underline{C}} = \underline{v}_i^T \qquad (i = 1,2,\ldots,\ell) \tag{2.23}$$

and task 3 can only be fulfilled if $\underline{r}_i$ satisfies

$$\underline{\underline{B}}\,\underline{r}_i = \underline{w}_i \qquad (i = 1,2,\ldots,\ell) \tag{2.24}$$

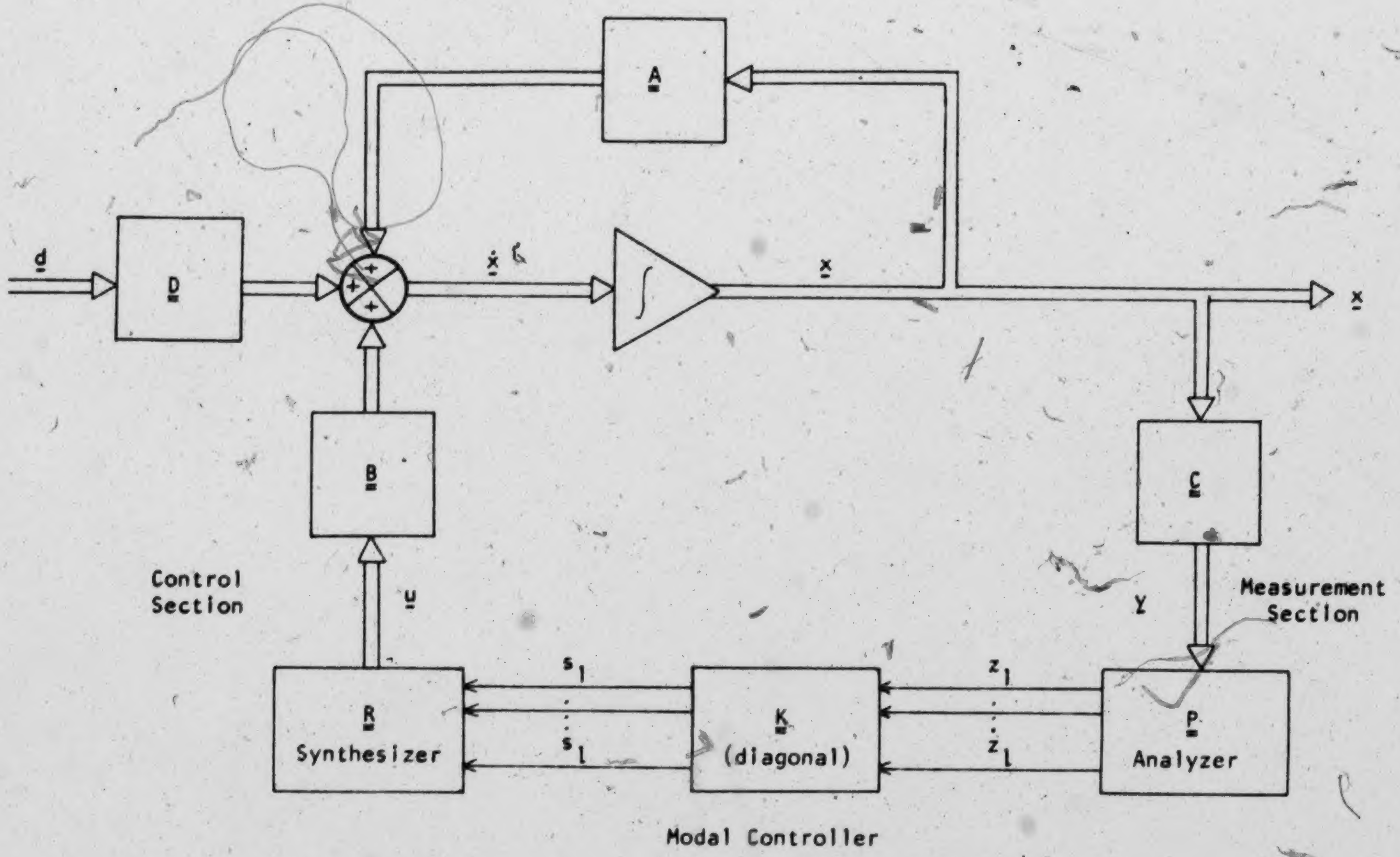

Figure 2.1 Ideal Modal Control

where $\underline{p}_i^T$ is the i^{th} row of $\underline{\underline{P}}$, $\underline{r}_i$ is the i^{th} column of $\underline{\underline{R}}$, and ℓ is any positive integer less than or equal to n [4]. Thus, if all the modes of the system are to be controlled the following relations must hold:

$$\underline{\underline{P}}\ \underline{\underline{C}} = \underline{\underline{V}}^T \tag{2.25}$$

and

$$\underline{\underline{B}}\ \underline{\underline{R}} = \underline{\underline{W}}\ . \tag{2.26}$$

However, if only ℓ modes are to be controlled, then ℓ columns of the $\underline{\underline{R}}$ matrix are chosen such that the first ℓ columns of $\underline{\underline{B}}\ \underline{\underline{R}}$ are the ℓ right eigenvectors corresponding to the ℓ modes, and the ℓ rows of $\underline{\underline{P}}$ are chosen such that the first ℓ rows of $\underline{\underline{P}}\ \underline{\underline{C}}$ are the corresponding left eigenvectors.

Under the assumption of the availability of n independent measurements and controls, (2.23) and (2.24) represent n equations in n unknowns, namely the elements of the $\underline{p}_i^T$ and $\underline{r}_i$ vectors. But, when the number of measurements and controls are less than n, it is in general impossible to satisfy (2.23) and (2.24) exactly, because they now represent n equations in m and r unknowns, respectively. Thus the modal activation of a particular mode cannot be detected, and the state vector cannot be driven in the opposite direction of that particular modal activation.

The feedback control law used in Figure 2.1 is of the general form:

$$\underline{u} = -\underline{\underline{G}}\ \underline{y}\ . \tag{2.27}$$

Considering (2.25) and (2.26) together with Figure 2.1, (2.27) can be written explicitly as [6]:

$$\underline{u} = - \underline{\underline{B}}^{-1} \underline{\underline{W}}\, \underline{\underline{K}}\, \underline{\underline{V}}^T \underline{\underline{C}}^{-1} \underline{y} \quad . \tag{2.28}$$

Combining this last equation with (2.1) and (2.2), the closed-loop system representation in the state space is obtained:

$$\dot{\underline{x}}(t) = (\underline{\underline{A}} - \underline{\underline{W}}\, \underline{\underline{K}}\, \underline{\underline{V}}^T)\underline{x}(t) + \underline{\underline{D}}\, \underline{d}(t) \quad . \tag{2.29}$$

In the case where $\underline{\underline{A}}$ has distinct eigenvalues, $\underline{\underline{A}}$ can be written as,

$$\underline{\underline{A}} = \underline{\underline{W}}\, \underline{\underline{\Lambda}}\, \underline{\underline{V}}^T \tag{2.30}$$

and (2.29) can be written as:

$$\dot{\underline{x}}(t) = \underline{\underline{W}}(\underline{\underline{\Lambda}} - \underline{\underline{K}})\, \underline{\underline{V}}^T \underline{x}(t) + \underline{\underline{D}}\, \underline{d}(t) \quad . \tag{2.31}$$

This last equation clearly shows that the modal controller has effectively created a new system with a new set of eigenvalues, λ_{di}, given by,

$$\lambda_{di} = \lambda_i - k_i \quad (i = 1,2,\ldots,n) \tag{2.32}$$

but with the same right and left eigenvector matrices, $\underline{\underline{W}}$ and $\underline{\underline{V}}^T$. In (2.32) λ_{di} represents the i^{th} closed-loop eigenvalue and k_i, the i^{th} diagonal element of the diagonal $\underline{\underline{K}}$ matrix.

In other words, for $k_i > 0$, the time constants associated with the modes have been reduced, but the directions of the modes have remained unchanged. This is the net effect of the ideal modal controllers on the system eigenproperties.

The effect of the ideal modal controllers on the system response can be demonstrated by considering a step change of inputs at time zero. For the case where ℓ of the n modes are controlled the response of the closed-loop system can be obtained from (2.22) [4]:

$$\underline{x}(t) = \sum_{i=1}^{\ell} - \frac{\underline{v}_i^T \underline{D}\, \underline{d}}{\lambda_i - k_i} [1 - e^{(\lambda_i - k_i)t}]\underline{w}_i$$

$$+ \sum_{i=\ell+1}^{n} - \frac{\underline{v}_i^T \underline{D}\, \underline{d}}{\lambda_i} (1 - e^{\lambda_i t})\underline{w}_i \; . \qquad (2.33)$$

Obviously, the factors $(\frac{1}{\lambda_i - k_i})$ in the first summation reduce the i^{th} activation to a great extent for large values of k_i. Thus, for inputs driving the process away from steady state, the ideal modal controller will decrease the magnitude of the undesirable contributions of the ℓ controlled modes to the system response.

Another interesting property of the modal controllers can be depicted by rewriting (2.31) as [6]:

$$\dot{\underline{x}}(t) = \underline{A}\,\underline{x} - \sum_{i=1}^{\ell} k_i <\underline{v}_i, \underline{x}> \underline{w}_i + \underline{D}\,\underline{d} \qquad (2.34)$$

or

$$\dot{\underline{x}}(t) = \underline{A}\,\underline{x} - \sum_{i=1}^{\ell} k_i \underline{E}_i \underline{x} + \underline{D}\,\underline{d} \qquad (2.35)$$

where $\underline{E}_i = \underline{w}_i><\underline{v}_i$ denotes the dyadic product of the right and left eigenvectors corresponding to the i^{th} mode. Thus, ideal modal controllers represent the sum of a set of dyadic single mode controllers, namely, $\underline{u} = \sum_{k_i}^{\ell} k_i \underline{E}_i \underline{x}$. Associated with each mode to be controlled there is a measurement vector, $\underline{v}_i$, a manipulation vector, $\underline{w}_i$, and a gain k_i, which directly reflects the amount by which the corresponding open-loop eigenvalue has been changed.

The modal control, through the use of modal analysis and synthesis, can be employed to shift each open-loop system eigenvalue indpendently to a prespecified closed-loop location. Eigenvalue assignment has a similar aim: replacement of the open-loop eigenvalue set with a more desirable set for the closed-loop system. In those instances where the designer has access to all the values of the system state variables, modal analysis has been employed in eigenvalue assignment to gain control over the fate of each individual eigenvalue.

Modal control changes the system eigenvalues by controlling its modes while eigenvalue assignment changes the system eigenvalues without directly controlling its modes.

2.4 Use of Ideal Modal Analysis in Eigenvalue Assignment

The control method to be described in this section is mainly due to Simon and Mitter [7]. It performs two of the tasks of ideal modal control. It determines the modal state variables and uses feedback to assign the system eigenvalues without directing the controls along the right eigenvectors. This has several implications:

1) Some or all of the system eigenvalues can be positioned without changing the remaining ones. The necessary and sufficient condition required is the complete controllability of the modes whose eigenvalues are to positioned.

2) The modes will be interacting and at least some of the eigenvectors of the system will change direction arbitrarily.

3) The control energy required for a given eigenvalue shift is greater than that required by an ideal modal controller [8].

4) Measurement of all of the state variables is necessary, but the number of controls, r, can be less than the number of states.

The theoretical basis of modal analysis and eigenvalue assignment has been presented by Simon and Mitter [7], Wonham [9], and Mitter and Foulkes [10]. Two extremely useful design tools in the eigenvalue assignment problem have been introduced by Simon and Mitter [7]; the dyadic controller and the recursive design technique. Since these tools have been extensively used in subsequent studies, Simon and Mitter's work [7] deserves careful consideration. In the following discussion it will be assumed that the open-loop system has distinct eigenvalues or has been converted into one with distinct eigenvalues as described in [11].

Three basic design steps are involved in Simon and Mitter's work [7]:

i) Conversion of the system equations into their modal canonical form via a similarity transformation.

ii) Construction of the constant controller matrix providing for the desired eigenvalue movements in the modal space.

iii) Conversion of this controller into the form $\underline{u} = \underline{\underline{G}}\,\underline{x}$.

The system representation obtained through the application of the first step will be that given by (2.10). Next, define the control law to shift ℓ of the system eigenvalues ($\ell \leq n$) by feeding back the corresponding ℓ modal state variables

$$\underline{u} = \tilde{\underline{\underline{G}}}\,\underline{z}_\ell \tag{2.36}$$

or equivalently

$$\underline{u} = \tilde{\underline{g}}_1 z_1 + \tilde{\underline{g}}_2 z_2 + \dots + \tilde{\underline{g}}_\ell z_\ell \tag{2.37}$$

where $\tilde{\underline{g}}_1, \tilde{\underline{g}}_2, \dots, \tilde{\underline{g}}_\ell$ are r-dimensional vectors representing the first ℓ columns of $\tilde{\underline{\underline{G}}}$. The remaining $(n-\ell)$ columns consist of $\underline{0}$-vectors.

The resulting closed-loop system in the modal domain is given by

$$\dot{\underline{z}}(t) = (\underline{\underline{\Lambda}} + \underline{\underline{H}}\,\tilde{\underline{\underline{G}}})\underline{z} + \underline{\underline{E}}\,\underline{d}(t) \qquad (2.38)$$

or equivalently [7]:

$$\dot{\underline{z}}(t) = \left[\begin{array}{cccc|cc}
\lambda_1+\alpha_{11} & \alpha_{12} & \cdots & \alpha_{1\ell} & & \\
\alpha_{21} & \lambda_2+\alpha_{22} & \cdots & \alpha_{2\ell} & & \\
\alpha_{31} & \alpha_{32} & \cdots & \alpha_{3\ell} & \underline{\underline{0}} & \\
\vdots & \vdots & & \vdots & & \\
\alpha_{\ell} & \alpha_{\ell} & \cdots & \lambda_{\ell}+\alpha_{\ell\ell} & & \\
\hline
\alpha_{\ell+1,1} & \alpha_{\ell+1,2} & \cdots & \alpha_{\ell+1,\ell} & \lambda_{\ell+1} & \underline{\underline{0}} \\
\vdots & \vdots & & \vdots & & \\
\alpha_{n1} & \alpha_{n2} & \cdots & \alpha_{n\ell} & \underline{\underline{0}} & \lambda_n
\end{array}\right]\underline{z}(t)+\underline{\underline{E}}\,\underline{d}(t) \qquad (2.39)$$

where

$$\alpha_{ij} = \underline{h}_i^T\,\tilde{\underline{g}}_j \quad (i = 1,2,\ldots,n;\ j = 1,2,\ldots,\ell) \qquad (2.40)$$

and $\underline{h}_i^T$ represents the i^{th} row of $\underline{\underline{H}}$.

Equation (2.39) indicates that the last $(n-\ell)$ eigenvalues of the system are unaffected by the selected control scheme. On the other hand, the first ℓ eigenvalues can be assigned by choosing $\tilde{\underline{g}}_1, \tilde{\underline{g}}_2, \ldots, \tilde{\underline{g}}_\ell$ such that the upper left partition of the system matrix in (2.39), denoted by $\bar{\underline{\underline{\Lambda}}}$, has the desired eigenvalues. This can be achieved by the following 3 steps:

1) Find the characteristic equation of $\bar{\underline{\underline{\Lambda}}}$:

$$\det [s\underline{\underline{I}} - \bar{\underline{\underline{\Lambda}}}] = s^{\ell} + a_1 s^{\ell-1} + \ldots + a_{\ell-1}s + a_{\ell} = 0 \ . \qquad (2.41)$$

Note that (2.39) and (2.41) imply that, in general, a_i $(i = 1,2,\ldots,\ell)$ are nonlinear functions of the components of $\tilde{\underline{g}}_i$ $(i = 1,2,\ldots,\ell)$. But, in the important special case where $r = 1$, they are linear functions of the $\tilde{g}_i$'s.

2) Obtain the characteristic equation corresponding to the desired eigenvalue set, $\{\lambda_{d1}, \lambda_{d2}, \ldots, \lambda_{d\ell}\}$:

$$(s-\lambda_{d1})(s-\lambda_{d2})\ldots(s-\lambda_{d\ell}) = s^{\ell}+b_1 s^{\ell-1}+\ldots+b_{\ell-1}s+b_\ell \ . \qquad (2.42)$$

3) Compare the two characteristic equations of Steps 1 and 2. For the closed-loop system to have the desired eigenvalues, the following condition must hold:

$$a_i = b_i \quad (i = 1,2,\ldots,\ell) \ . \qquad (2.43)$$

Equation (2.43) consists of a set of ℓ nonlinear equations in the $(\ell \cdot r)$ unknown components of $\tilde{\underline{\underline{G}}}$.

Dyadic Approach:

The above discussion reveals the difficult computational problem involved in the method and suggests a possible solution to it, namely conversion of the multi-input system to an equivalent single-input system in order to convert a nonlinear algebraic problem to a linear one. This idea, which forms the basis of the dyadic approach, is realized by restricting the controller to have linearly dependent columns. Thus the suggested new control law has the form [7]:

$$\underline{u} = [k_1\underline{g},\ k_2\underline{g},\ \ldots,\ k_\ell\underline{g}]\underline{z}_\ell \qquad (2.44)$$

or simply

$$\underline{u} = \underline{g}\ \underline{k}^T\ \underline{\underline{V}}_\ell^T\ \underline{x} \qquad (2.45)$$

if it is desired to assign ℓ of the system eigenvalues corresponding to the ℓ modal state variables, $\underline{z}_{\ell}$. $\underline{\underline{V}}_{\ell}^{T}$ contains the ℓ left eigenvectors corresponding to the eigenvalues to be shifted, as its ℓ rows.

The equivalent single input system to that expressed by (2.1) is thus given as:

$$\dot{\underline{x}}(t) = \underline{\underline{A}}\,\underline{x}(t) + \underline{b}\,u(t) + \underline{\underline{D}}\,\underline{d}(t) \tag{2.46}$$

where

$$\underline{b} \equiv \underline{\underline{B}}\,\underline{g} \tag{2.47}$$

and

$$u(t) \equiv \underline{k}^{T}\,\underline{\underline{V}}_{\ell}^{T}\,\underline{x} \quad . \tag{2.48}$$

The choice of $\underline{g}$ is arbitrary as long as the equivalent system in (2.46) remains controllable, i.e.,

$$\underline{h}_{i}^{T}\,\underline{g} \neq 0 \quad (i = 1,2,\ldots,n) \quad . \tag{2.49}$$

The choice of $\underline{k}^{T}$, on the other hand, is dictated by the desired set of eigenvalues. This problem has been considered by Simon and Mitter [7], Gould, Murphy and Berkman [12], and Retallack and MacFarlane [13]. The solution, which seems to be the most straightforward and computationally easy one [13], is based on a result of Hsu and Chen [14] relating the ratio of the closed- and open-loop characteristic equations to the determinant of the system return-difference matrix.

The elements of $\underline{k}^{T}$ are given by [13]:

$$k_{i} = \frac{\xi_{i}}{\beta_{i}} \, , \quad (i = 1,2,\ldots,\ell) \tag{2.50}$$

where

$$\beta_{i} = \underline{v}_{i}^{T}\,\underline{\underline{B}}\,\underline{g} \tag{2.51}$$

and

$$\xi_i = \frac{\prod_{j=1}^{\ell} (\lambda_i - \lambda_{dj})}{\prod_{\substack{j=1 \\ j\neq i}}^{\ell} (\lambda_i - \lambda_j)} \qquad (2.52)$$

Design Freedom:

The arbitrariness in the choice of $\underline{g}$ can be utilized as a degree of freedom for one of the following design objectives:

1) eliminating the need for certain controls,

2) eliminating the need for certain measurements,

3) maximizing the "measure of controllability" of the shifted eigenvalues thus minimizing the absolute value of the controller gains to be used,

4) affecting the transient response of the system.

This first objective can be realized by simply assigning the value of zero to the elements of $\underline{g}$ corresponding to the control elements to be neglected providing that the controllability condition given by (2.49) is not violated.

Simon and Mitter [7] have suggested that $(r-1)$ of the n state measurements can be deleted without affecting the attainability of n eigenvalue locations. This can be achieved by choosing the gain elements in $\underline{k}^T$ such that the row vector $(\underline{k}^T \underline{\underline{V}}_\ell^T)$ defined in (2.48) has $(r-1)$ zero elements. These zero elements will correspond to those state variables, which now do not have to be measured. Then $\underline{g}$ can be determined to assign the desired eigenvalue set.

The third objective is of great practical importance, since high controller gains may lead to the unsatisfactory control of a real process due to noise effects, model inaccuracies, nonlinearities, and saturation of manipulated variables. In the event of shifting one system eigenvalue, say λ_1, by using one control element only, a measure of the degree of controllability of λ_1 by the scalar control u considered is given by [7]:

$$\psi_1 = \left| \frac{\langle \underline{v}_1, \underline{b} \rangle}{\langle \underline{v}_1, \underline{v}_1 \rangle^{\frac{1}{2}}} \right| \tag{2.53}$$

where $\underline{v}_1$ is the left eigenvector corresponding to λ_1 and $\underline{b}$ is the control coefficient vector corresponding to the control element u. ψ_1 has been generalized by Simon and Mitter [7] to the case of dyadic control using all the control elements by:

$$\psi_1 = \left| \frac{\underline{v}_1^T \underline{\underline{B}}\, \underline{g}}{\langle \underline{v}_1, \underline{v}_1 \rangle^{\frac{1}{2}} ||\underline{g}||} \right| \tag{2.54}$$

where $||\underline{g}|| = \max\limits_{i=1,\dots,r} |g_i|$ by definition.

The above measure of controllability is maximized through the choice of $\underline{g}$ as [7]:

$$g_i = \text{sign}\,(\underline{v}_1^T \underline{b}_i) \quad (i = 1,2,\dots,r) \quad . \tag{2.55}$$

The concepts of the degree of controllability and the degree of observability have been considered for multi-input multi-output systems in [15-18]. But the problem of adjusting the variable system parameters, e.g., $\underline{g}$, such that these degrees are maximized seems to offer challenging mathematical difficulties [17, 18].

The fourth point is also of great practical importance. Several authors [19-22] have already noted that for multi-input systems the controller matrix assigning a given set of eigenvalues is non-unique.

In particular each choice of $\underline{g}$ will result in a different controller matrix. Thus, in general the eigenvectors of the closed-loop system will depend on the choice of $\underline{g}$ which implies that for the same set of closed-loop eigenvalues, different transient reponses will be obtained. The analytical relation between the direction of $\underline{g}$ and the closed-loop eigenvector directions, and thus the shape of the transient response, is not known.

The dyadic approach above described simplifies the computational aspect of eigenvalue assignment, but this advantage is gained at the cost of losing some design freedom in specifying $\underline{\underline{G}}$ [13, 21]. The recursive design technique of Simon and Mitter [7] offers a solution to this problem and also reduces the computational effort if only a subset of eigenvalues are to be assigned.

The recursive dyadic approach of Simon and Mitter [7] consists of a series of steps where at each step a small subset of eigenvalues are shifted by the single-step dyadic approach and the closed-loop system matrix resulting from the last step is considered to be the new open-loop system matrix. Assignment of ℓ eigenvalues by this approach, one at a time, will lead to the following control law:

$$\underline{u} = \underline{u}_1 + \underline{u}_2 + \dots + \underline{u}_\ell \tag{2.56}$$

where

$$\underline{u}_i = \underline{g}_i \, k_i \, \underline{v}_i^T \underline{x} \qquad (i = 1,2,\dots,\ell) \tag{2.57}$$

and

$$k_i = \frac{\lambda_{di} - \lambda_i}{\langle \underline{\underline{B}}^T \underline{v}_i , \underline{g}_i \rangle} \qquad (i = 1,2,\dots,\ell) \ . \tag{2.58}$$

The determination of $\underline{g}_i$ to satisfy some design objectives in addition to eigenvalue assignment is an unresolved problem in control systems theory.

Simon and Mitter's [7] work has been very useful from a theoretical point of view, and has provided the basis for more practical methods, which attempt to assign only a small subset of system eigenvalues and require only a small number of measurements. These methods are discussed in Sections 2.5, 2.7-2.10, and in Chapter Three.

2.5 When the Number of Measurements and the Number of Controls are Less than the Number of States

The application of ideal modal control requires the availability of n independent measurements and n independent controls, but in industrial situations this requirement usually cannot be fulfilled due to either physical or economical considerations. Thus, the problem of conditioning the system modes in the absence of n independent measurements and controls is an important one.

Measurement of a small number of state variables leads to the projection of the state vector onto a subspace of the state space, called the measurement space. To determine the modal states, one must be able to measure the components of the state vector along the right eigenvectors (cf., (2.4), (2.5)). This is done by determining the projection of the state vector along the corresponding right eigenvector. But the deficiency in dimension of the measurement space prohibits this and leads one to determine the projections of the right eigenvectors onto the measurement space instead. These projections are then used in place of the right eigenvectors. The resulting set of vectors, $(\underline{w}_1^*, \underline{w}_2^*, \ldots, \underline{w}_n^*)$, is linearly dependent. Consequently, the projection of the left eigenvector of the i^{th} controlled mode onto the measurement space, namely $\underline{v}_i^*$, will not be in general orthogonal to the set,

$\underline{w}_1^*, \ldots, \underline{w}_{i-1}^*, \underline{w}_{i+1}^*, \ldots, \underline{w}_n^*$. Thus, measuring the activation of a controlled mode will, in general, result in a quantity that contains the activation of all the modes.

The activation of a mode can be estimated reasonably well by the activation of the mapping of the mode in the measurement space if the components of the mode that are orthogonal to the measurement space are small in magnitude compared to the mode itself. If any mode is orthogonal to the measurement space, then that mode is completely unobservable. Therefore, the measurement space must be chosen such that the mode whose activation is to be measured has small orthogonal components.

In ideal modal control if only one mode is disturbed, then the resultant control action is directed along the appropriate right eigenvector, so that the component added to the state velocity vector, $\dot{\underline{x}}(t)$, directly counters the disturbance. Thus, the control action does not influence the other modes. This cannot be achieved if the rank $\underline{\underline{B}} = r < n$, since then the manipulation space spanned by the columns of $\underline{\underline{B}}$ cannot, in general, contain the appropriate right eigenvector. In fact, generally all of the right eigenvectors will be outside of the manipulation space. It is also worth noting that, if a mode is orthogonal to the manipulation space then that mode is uncontrollable.

In summary, the small number of measured variables means that estimates of modal activations cannot be the true activations, and the small number of manipulated variables means that the control action cannot exactly counteract the activations of the controlled modes. As a result the control action produces disturbances in modes other than the controlled mode [4, 6].

Two basically different but practically quite similar approaches have been adopted to cope with these difficulties, approximate modal control and eigenvalue assignment techniques.

The major reasons for the inability to achieve ideal modal control in the absence of n independent measurements and controls are the difficulties involved in estimating a specific modal activation and in influencing a specific mode without causing any modal interactions. Consequently, there is considerable motivation for trying to alleviate these difficulties in order to achieve some of the favorable results obtainable from ideal modal control. This approach will be classified here as "approximate modal control".

The major effect of modal control, which accounts for its success, is the shift of the open-loop eigenvalues to more favorable locations in the closed-loop system. Thus, one should try to change the system eigenvalues disregarding any interaction between the modal states which will result from this practice. This approach will be classified here as eigenvalue assignment. Eigenvalue assignment techniques simply insure that some set of desired eigenvalues form a subset of the closed-loop system eigenvalues. Thus, they do not provide any pairing between open-loop and closed-loop eigenvalues, and cannot control the movement of the unassigned set of system eigenvalues. Nor do they attempt to preserve the original eigenvectors.

Eigenvalue assignment methods using approximate modal analysis try to pair open-loop and closed-loop eigenvalues and to maintain the unassigned set of eigenvalues at their open-loop locations.

Several successful simulation studies employing approximate modal control have been reported. This is mainly due to the fact that modal control provides the designer with criteria for deciding which modes to control and which measurements and manipulated variables to use for this purpose. The success of the resulting modal controller largely depends how well one can meet these criteria. Eigenvalue assignment, on the other hand, does not provide the designer with direct information related to the control configuration that should be used. But, it is possible to combine modal analysis with eigenvalue assignment, and this in fact leads to what has been classified as "approximate modal analysis" in the preceding paragraph. Approximate modal analysis as developed by E.J. Davison and his co-workers [23-25] is, in the author's opinion, one of the more practical and powerful techniques described in this thesis. There are, of course, special applications in which some of the other techniques will prove to be more useful.

2.6 Criteria for the Selection of Dominant Modes, Measurements, and Controls

Physical systems with a few well-separated dominant modes have been reported to perform better under modal control and eigenvalue assignment schemes using modal analysis than under conventional control schemes [1, 4, 6, 23-28]. Thus, the designer should first analyze the system modes to determine the dominant ones, namely, the ones which affect the system response the most. This is extremely important in view of the fact that the control of every additional mode results in much greater difficulties in the design stage.

Consideration of (2.20) indicates that there are three attributes associated with a mode which must be analyzed:

1) The right eigenvector corresponding to the mode. Comparison of the i^{th} element of a normalized right eigenvector to the i^{th} element of the other right eigenvectors indicates the relative influence of that mode on the i^{th} state variable [29].

2) The eigenvalue of the mode. This indicates the time constant associated with the mode.

3) The modal activations. These indicate the influence of a mode on a state variable for specific disturbances and initial conditions.

The first two attributes are the more important ones to consider. It is desirable to control those modes which have the largest influence on the important process variables and among these, the modes with the larger time constants. In case some of the modes are unstable, they must definitely be included among those to be controlled.

Some typical processes have a wide spectrum of eigenvalues, thus the choice of the modes to be controlled might seem easy. But it is quite possible that the occurence of a disturbance will create dominant modes out of less important ones by highly activating them.

The next question which needs some consideration involves the choice of the "best" measurements and manipulated variables from the often restricted set of possible ones in order to control the dominant modes.

The "best" measurements, in the modal sense, are those which are most sensitive to the controlled modes and least sensitive to the uncontrolled modes. The "best" controls are those which have the most effect on the controlled modes while having the least effect on the uncontrolled ones.

The above definitions are applicable only if the measurement and control sections of the modal controller are noninteractive. This being true for the ideal modal controllers only, the following criteria for the choice of the measurement and control spaces are only approximately true:

1) The measurement space should be chosen such that components of the controlled modes which are orthogonal to it are of minimal magnitude and the projections of the uncontrolled modes onto this space have minimal length.

2) The control space should be chosen such that the projections of its basis vectors on the controlled modes are of maximal length while being of minimal length on the uncontrolled modes.

These arguments are another manifestation of the desirability of forming measurement and control sections, which in the absence of n independent measurements and controls, act like the left and right eigenvector sets of the original system.

2.7 Approximate Modal Control

Approximate modal control aims at changing the dominant eigenvalues of a system, which has fewer than n measurements and n controls, without changing any of its other eigenproperties. Basically, the problem consists of designing measurement, $\underset{\sim}{P}\,\underset{\sim}{C}$ and control, $\underset{\sim}{B}\,\underset{\sim}{R}$, sections, which in the absence of n measurements and n controls, play the same role as their counterparts do in an ideal modal controller described in Section 2.3. Two different approaches have been used in attacking this problem:

1) On the basis of the criteria given in Section 2.6, choose the "best" measurement and control variables, thus fixing the $\underline{C}$ and $\underline{B}$ matrices. Then, determine the $\underline{P}$ and $\underline{R}$ matrices in one of the alternative ways to be described later.

2) Rather than first choosing the "best" measurement and control variables and then determining the measurement and control sections, perform these two tasks simultaneously.

The first approach has been adopted by Rosenbrock [1], and by Takahashi and his co-workers [5, 30]. In describing these methods it will be assumed for simplicity that the number of measurements and controls are equal $(m = r = \ell)$ and the open-loop eigenvalues are distinct, the generalization of the methods for the case where $m \neq r$ is straightforward and can be found in the original works [1, 5, 30].

Rosenbrock [1] suggests choosing the $\underline{P}$ matrix in Figure 2.1 as

$$\underline{P} = (\underline{C}\,\underline{W}_{\ell})^{-1} \qquad (2.59)$$

where $\underline{W}_{\ell}$ contains as its columns the ℓ right eigenvectors corresponding to the ℓ dominant modes of the system. Similarly, the $\underline{R}$ matrix in Figure 2.1 should be determined from

$$\underline{R} = (\underline{V}_{\ell}^{T}\,\underline{B})^{-1} \qquad (2.60)$$

where $\underline{V}_{\ell}^{T}$ contains as its rows the ℓ left eigenvectors belonging to the ℓ dominant modes of the system. Thus, the approximate modal controller designed by Rosenbrock's method becomes:

$$\underline{G} = (\underline{V}_{\ell}^{T} \underline{B})^{-1} \underline{K}(\underline{C} \underline{W}_{\ell})^{-1} \tag{2.61}$$

where $\underline{K}$ is an $\ell \times \ell$ diagonal matrix, whose diagonal elements are to be obtained from (2.32). This method has been applied in simulation studies to the control of distillation columns [1, 4, 26, 28] and to the control of a boiler [27]. In all of these studies it has been observed that the control of a very small number of dominant modes gave a significant improvement over that obtainable by corresponding conventional controllers. Levy's work [4], which is mainly concerned with the modal analysis of different distillation column models, is extremely useful for any further work in this area. It effectively demonstrates the usefulness of modal analysis in determining the control configuration of a system.

Takahashi et al [30] have realized that the objectives of approximate modal control as defined at the beginning of this chapter could be realized if the $(n-\ell)$ uncontrolled modes of the system were uncontrollable and unobservable while the remaining ℓ modes are both controllable and observable. For the type of systems under consideration this would imply that the $\underline{H}$ and $\underline{F}$ matrices defined by (2.13) and (2.15) are of the following special form:

$$\underline{H} = \begin{bmatrix} \underline{H}_{\ell} \\ \hdashline \underline{\Delta H} \end{bmatrix} = \begin{bmatrix} \underline{H}_{\ell} \\ \hdashline \underline{0} \end{bmatrix} \tag{2.62}$$

$$\underline{F} = [\underline{F}_{\ell} \mid \underline{\Delta F}] = [\underline{F}_{\ell} \mid \underline{0}] \quad . \tag{2.63}$$

Any system possessing this characteristic has been designated by Takahashi et al [28] as one with "ideal measurement and control" sections.

The corresponding modal controller is again the one given by (2.61). If (2.62) and (2.63) are fulfilled, this controller performs the functions of an ideal modal controller for the ℓ modes of the system provided that

$$\text{rank } \underline{\underline{H}}_\ell = \text{rank } \underline{\underline{F}}_\ell = \ell \; . \qquad (2.64)$$

If, on the other hand, only (2.62) and (2.64), or (2.63) and (2.64) hold, then the values of ℓ of the system eigenvalues can be changed without changing the values of the remaining eigenvalues, but nothing can be said about the directions of the closed-loop eigenvectors.

The conditions for ideal control and measurement sections as expressed by (2.62) and (2.63) are very difficult to satisfy. The failure to fulfill either (2.62) or (2.63) implies that the controller of (2.61) cannot exactly fulfill any of the objectives of an ideal modal controller. This can be demonstrated by considering the modal domain closed-loop system matrix:

$$\left[\begin{array}{c|c} \underline{\underline{\Lambda}}_{\ell d} & -\underline{\underline{K}}\,\underline{\underline{F}}_\ell^{-1}\,\underline{\underline{\Delta F}} \\ \hline -\underline{\underline{\Delta H}}\,\underline{\underline{H}}_\ell^{-1}\,\underline{\underline{K}} & \underline{\underline{\Lambda}}_{n-\ell} - \underline{\underline{\Delta H}}\,\underline{\underline{H}}_\ell^{-1}\,\underline{\underline{K}}\,\underline{\underline{F}}_\ell^{-1}\,\underline{\underline{\Delta F}} \end{array}\right] \qquad (2.65)$$

where $\underline{\underline{\Lambda}}_{\ell d}$ is an $\ell \times \ell$ diagonal matrix with diagonal entries containing the ℓ desired eigenvalues of the closed-loop system. $\underline{\underline{\Lambda}}_{n-\ell}$ is an $(n-\ell) \times (n-\ell)$ matrix containing the $(n-\ell)$ open-loop eigenvalues corresponding to the uncontrolled modes. Thus, the performance of the controller of equation (2.61) will depend on the designer's ability to find such measurements and controls which minimize the elements of $\underline{\underline{\Delta F}}$ and $\underline{\underline{\Delta H}}$.

Loscutoff [5] has suggested specifying nonzero off-diagonal elements in $\mathbf{K}$ such that either $\mathbf{K}\,\mathbf{F}^{-1}\,\Delta\mathbf{F}$ or $\Delta\mathbf{H}\,\mathbf{H}_{\ell}^{-1}\,\mathbf{K}$ would become a null matrix without affecting the assignability of ℓ eigenvalues. This approach necessitates the solution of nonlinear algebraic equations. Also, it cannot be employed unless the condition, $m+r \geq n+1$, holds. Whenever this last inequality holds all of the n system eigenvalues can be assigned. This approach does not provide any improvement over Simon and Mitter's [7] suggestion to reduce the number of state measurements (see Section 2.4), in cases where the state variables rather than the outputs are measured.

Apart from their contribution to modal analysis, Takahashi et al [30] have suggested the derivation of another approximate modal controller, whose analyzer and synthesizer are obtained from (2.25) and (2.26) by using right and left pseudoinverses. Thus,

$$\mathbf{P} = \mathbf{F}^T(\mathbf{F}\,\mathbf{F}^T)^{-1} \tag{2.66}$$

$$\mathbf{R} = (\mathbf{H}^T\,\mathbf{H})^{-1}\,\mathbf{H}^T \tag{2.67}$$

and the resulting controller is given by

$$\mathbf{G} = (\mathbf{H}^T\,\mathbf{H})^{-1}\,\mathbf{H}^T\mathbf{K}\,\mathbf{F}^T(\mathbf{F}\,\mathbf{F}^T)^{-1} \tag{2.68}$$

where $\mathbf{K}$ is an $n \times n$ diagonal matrix, whose diagonal elements are obtained from (2.32) of this chapter. The existence of the pseudoinverses in equations (2.66) and (2.67) depend on the following rank conditions

$$\text{rank } \mathbf{F} = \text{rank } \mathbf{H} = \ell\ . \tag{2.69}$$

Takahashi et al [30] have not supplied any physical or mathematical conditions under which this method would perform satisfactorily. This is the major weakness of the method; one does not have any a priori knowledge about the expected performance of the controller.

The second approach mentioned at the beginning of this chapter has been developed by Howarth [6], who has also applied the method to control one or two modes of the distillation column models developed in [4] and [28].

Howarth has defined two objective functions, which give a quantitative measure of the functional resemblance of the measurement and control sections to be synthesized to those of an ideal modal controller. The objective function defined for the measurement section is

$$\alpha = \max_{j \neq i} \left| \frac{\langle \underline{\underline{C}}^T \, \underline{p}_i^T , \underline{w}_j \rangle}{\langle \underline{\underline{C}}^T \, \underline{p}_i^T , \underline{w}_i \rangle} \right| \qquad (j = 1,2,\ldots,n) \qquad (2.70)$$

and that for the control section is

$$\beta = \max_{j \neq i} \left| \frac{\langle \underline{v}_j , \underline{\underline{B}} \, \underline{r}_i \rangle}{\langle \underline{v}_i , \underline{\underline{B}} \, \underline{r}_i \rangle} \right| \qquad (j = 1,2,\ldots,n) \qquad (2.71)$$

where i refers to the controlled mode, $\underline{p}_i$ to the i^{th} row of $\underline{\underline{P}}$, $\underline{r}_i$ to the i^{th} column of $\underline{\underline{R}}$, where $\underline{\underline{P}}$ and $\underline{\underline{R}}$ have been defined in Figure 2.1. Minimization of α with respect to the possible measurements selectable from a fixed total set and with respect to the elements of $\underline{p}_i$ fixes the measurement section of the approximate modal controller for the i^{th} mode. Similarly, minimization of β with respect to the possible controls selectable from a fixed total set and with respect to the elements of $\underline{r}_i$ fixes the control section of the approximate modal controller for the i^{th} mode. Howarth [6] suggests attacking the multiple-mode control problem as a series of independent single-mode control problems, which can be solved by the above procedure. This is based on the assumption that the various single mode controllers thus designed will not interact in the modal domain. Apart from this the following assumptions have been made in [6]:

i) The measurement and control sections are noninteracting.

ii) The eigenvectors of the closed-loop system are the same as those of the open-loop system.

Howarth [6] has drawn several conclusions from his simulation studies involving the application of his technique to distillation column control:

1) For the case of single-mode control none of the eigenvalues of the system move towards less desirable locations. Thus, regardless of the gain values used, the natural stability of the system is preserved.

2) The controller approaches the performance of the ideal modal controller at low gain values.

3) The single-mode controller moves the zeros of the system close to the eigenvalues of the uncontrolled modes. This implies high sensitivity of the closed-loop system to parameter variations.

4) For single-mode controllers the eigenvalue of the controlled mode approaches that of the next slowest one as the value of the gain element is increased.

5) To approach the performance of an ideal modal controller for control of a single-mode, the eigenvalue of the slowest mode must be well separated from the other eigenvalues, and the eigenvectors must

be well distributed in the state space.

6) The performance of multiple-mode controllers sysnthesized by this approach is, generally, considereably less satisfactory than that of single-mode controllers.

Among the approximate modal controllers the one proposed by Howarth [6] comes closest in its performance to the ideal modal controller, but it is rather difficult to justify the computational efforts required in the design process. In fact, any modal controller which succeeds in reducing the time constants of the slow modes of the system and does not create any new slow modes may very well be worth considering. The identity of open- and closed-loop eigenvectors, which among the approximate controllers, is best achieved by Howarth's method, is a rather unjustified design criterion. It is possible for a controller to result in closed-loop eigenvectors which insure a better performance than the open-loop ones, e.g., better disturbance rejection properties, lower sensitivity to controller parameters, better steady state performance, etc.

2.8 Use of Approximate Modal Analysis in Eigenvalue Assignment

Davison and Goldberg [23] have combined modal analysis with the output feedback eigenvalue assignment technique of Davison [31] to create the output feedback counterpart of Simon and Mitter's [7] technique. It has been rather successfully applied in simulation studies by Davison and his co-workers [23-25].

Modal analysis involves determination of the modal state vector which, in general, involves the measurement of all of the state variables. Once some of the modal state variables are known, their proper feedback will ensure the assignment of the corresponding eigenvalues without disturbing the remaining ones. Thus, there is considerable incentive to determine the modal state variables. Davison and Goldberg's method [23] involves generating an approximation of the modal state variables via the measurement of a few state variables. This may be achieved by selecting those measurements which at normal operating conditions contribute most to the particular modal states whose eigenvalues are to be changed [23]. Now, feedback of m modal states, where m is the number of measurements, by use of the output feedback law devised in [31] will guarantee exact assignment of the m corresponding eigenvalues. Thus, if the modal state variables corresponding to the dominant modes can be approximated well enough, then the dominant eigenvalues will be changed. But, since in the absence of n measurements the modal states cannot be estimated exactly, the remaining $(n-m)$ eigenvalues will move from their open-loop locations, too.

The successful application of approximate modal analysis in eigenvalue assignment depends on how well the dominant eigenvalues are separated from the rest and how well one can estimate the modal states.

2.9 Exact and Approximate Eigenvalue Assignment via the Direct Use of the Process Measurements

Eigenvalue assignment techniques involve synthesis of a closed-loop system which possesses a specified set of eigenvalues. Unlike modal techniques, eigenvalue assignment techniques do not attempt to attribute a physical significance to each eigenvalue and are not directly concerned with the change in eigenvector directions.

Most of the well-known eigenvalue assignment techniques have adopted the objective of exactly attaining a specified closed-loop eigenvalue set. This objective seems to be an unnecessarily ambitious one at this time. The behavior of the closed-loop system depends on both its eigenvalue locations and its eigenvector directions, as indicated by (2.20). But, currently there is no systematic way of selecting these in order to guarantee certain performance characteristics. Also, the problem of synthesizing closed-loop eigenvectors has neither been solved nor attempted in the control literature.

A less ambitious objective employed in other eigenvalue assignment techniques is that of shifting the eigenvalues to some region in the complex plane rather than to specific locations. This region might be the left half of the complex plane or some region around a particular point. Techniques which try to achieve this objective will be referred to here as "approximate eigenvalue assignment techniques".

The process measurements are commonly all of the state variables, some subset of them or some functions of them. Feedback control techniques employing measurements of all of the state variables or n independent functions of them (i.e., the rank of matrix $\underline{C}$ in (2.2) is n, which in turn is the order of the system) are termed as state feedback techniques. Feedback control techniques employing some subset of the state variables are termed incomplete state feedback, and those employing m $(m < n)$ independent functions of them (i.e., rank $\underline{C} = m$) are termed output feedback techniques. Here, no distinction will be made between incomplete state and output feedback techniques, and they shall be referred to as output feedback techniques.

Most of the chemical processes which present control problems can be represented by high order models. It is also the rule, rather than the exception, that some of the state variables involved in these models cannot be measured directly with a desired amount of accuracy. In fact, economical considerations often put a stringent limit on the number of state variables that are measured. Thus, methods based on the assumption of availability of all the state variable measurements are not often of direct practical utility. Two alternatives available to the designer are:

1) Output feedback methods based on the original high order model.

2) State feedback methods based on a reduced order model.

Output feedback methods capable of handling both exact and approximate eigenvalue assignment have been devised. They are capable of fulfilling the eigenvalue assignment objective in cases where access to all of the state variable measurements is possible.

Model reduction techniques which are most appropriate for both modal control and eigenvalue assignment are those based on the modal characteristics of the system. Representative methods in this category can be found in [32-34] and numerous related articles. The success of model reduction in a particular system depends on how well the dominant modes retained in the reduced model are separated from the neglected modes. Use of model reduction in conjunction with modal control and exact eigenvalue assignment will almost invariably result in the approximate rather than exact assignment of the system eigenvalues.

The number of eigenvalues which can be assigned exactly in a particular system crucially depends on the number of available measurements and controls relative to the order of the model and on the number of controllable and observable modes. State feedback techniques guarantee the assignment of all those eigenvalues that belong to controllable modes [7, 9]. For output feedback techniques, on the other hand, the necessary and sufficient conditions for eigenvalue assignment are more stringent. Davison [31] and Davison and Chatterjee [35] have provided the sufficient conditions for the assignment of m and max (m,r) eigenvalues, respectively, where m and r are the number of nontrivial measurements and controls. The sufficient conditions for the assignment of (m+r-1) eigenvalues have been developed as part of this thesis and can be found in Chapter Three.

The more practical state feedback algorithms for eigenvalue assignment have been first developed by Simon and Mitter [7], Porter and co-workers [36-39], and by Anderson and Luenberger [40]. The algorithms in [7] and [36-39], which are very similar, have been derived from modal analysis. Their description has been attempted in Section 2.4. The extension of these methods to systems with complex eigenvalues can be found in [38-39]. Anderson and Luenberger's method [40] is based on the transformation of the system matrix to its phase variable canonical form. The linear transformation leading to the phase variable form (unlike the modal transformation) is nonunique for multi-input systems. This fact may be utilized as a design freedom [41, 42]. Also, this transformtion and the associated eigenvalue assignment methods do not necessitate the determination of the system eigenvalues and eigenvectors. These are a few advantages of Anderson and Luenberger's [40] method, but it has also disadvantages: The computational effort is almost independent of the number of eigenvalues to be placed, the system must be completely controllable or else decomposed via Kalman's structural theorem [43] before this eigenvalue assignment technique can be applied (this difficulty can be avoided by the use of the canonical form suggested in [44]), matrix inversions are required, and it does not possess the clarity and simplicity of Simon and Mitter's method [7] and its later modifications [12, 13].

The feedback matrix guaranteeing the assignment of a set of eigenvalues is not unique in multi-input systems. This design freedom can be exploited to fulfill other design criteria than assignment of a set of eigenvalues. In Simon's methods the choice of the $\underline{g}$ vector in (2.45), the pairing of open-loop and closed-loop eigenvalues, the number of eigenvalues shifted at each step of the recursive design, the sequence in which the open-loop eigenvalues are shifted and the number of control elements used at each step of the recursive design technique constitute the major design decisions. These design freedoms can be very important in obtaining a "better" transient and steady state response, a lower sensitivity to parameter variations, and a higher degree of integrity. But, it is unfortunate that systematic methods are currently not available to exploit these design freedoms, i.e., the designer has to tune the controller.

The more well-known output feedback techniques devised for exact eigenvalue assignment are due to Davison [31], Davison and Chatterjeee [35], and Jameson [45]. Sridhar and Lindorff [46] have provided an alternative constructive proof to Davison's theorem [31], but the validity of their proof has been questioned by Topaloglu and Seborg [47].

The techniques of [31, 45] ensure that the open-loop set of eigenvalues is replaced by a closed-loop set some of whose elements, the assigned eigenvalues, are prespecified while the remaining eigenvalues cannot be arbitrarily assigned. Experience with chemical process systems [4, 6, 23-28] indicates that changing the value of a few "badly" located dominant eigenvalues of a system can improve its performance significantly unless the rest of the eigenvalues move to "highly undesirable" locations. The eigenvalue assignment methods of [31, 35, 45] may not always fulfill these conditions since they do not ensure control over individual eigenvalues and cannot dictate the fate of the unassigned eigenvalues. Thus, it is very possible for a designer to end up with a closed-loop eigenvalue set which is worse than the open-loop one, because he has inadvertently moved the well-located eigenvalues to better positions and the "poorly" located eigenvalues being left uncontrolled have moved to worse locations.

The methods of [31, 35] and [45] can be improved by modifying them to be able to gain control over the individual eigenvalues, to increase the number of exactly assigned eigenvalues and to approximately assign the rest of the eigenvalues. The first objective may be achieved to some extent by the use of approximate modal analysis as described in Section 2.8. In cases where this approach can be applied satisfactorily, the controlled eigenvalues will be shifted to desirable closed-loop locations without shifting the uncontrolled eigenvalues. The second objective has been achieved to some extent through the algorithm developed as part of this thesis and described in Chapter Three. The third objective can be achieved by combining the approximate eigenvalue assignment techniques to be described next with the exact eigenvalue assignment techniques. This can be done by using a crucial design tool developed and employed in Chapter Three. Further information about this combined approach will be provided in the simulation study of Chapter Four.

The output feedback algorithm developed in Chapter Three is based on some of the ideas involved in [31, 35, 45]. In addition to enabling the designer to assign more eigenvalues, it provides additional design freedom through its recursive nature which in turn may lead to more successful applications. Since it includes the algorithms of [31, 35, 45] as special cases, the discussion of these algorithms will be omitted from this chapter.

Approximate Eigenvalue Assignment Techniques

Approximate eigenvalue assignment methods do not guarantee the exact assignment of any of the system eigenvalues, and have been devised mainly to stabilize unstable systems. General information on the subject of output feedback stabilization can be found in [48, 49], and necessary and sufficient conditions for system stabilizability by constant output feedback are provided in [50, 51].

For approximate eigenvalue assignment, Jameson [45] has suggested the use of a dyadic feedback controller, $\underline{G} = \underline{g}\,\underline{k}^T$, to minimize the value of the closed-loop characteristic polynomial, $r(\lambda)$, when it is evaluated at the desired eigenvalue locations. This can be done by selecting $\underline{g}$ to fulfill the condition in (2.49) and determining a $\underline{k}^T$ which will minimize the following objective function:

$$J_1 = \sum_{i=1}^{n} [r(\lambda_{di})]^2 \quad . \qquad (2.72)$$

Jameson [45] has generalized this objective function in order to weight the different eigenvalues according to their importance and to minimize the weighted sum of the squares of the elements of $\underline{k}^T$. This results in the following objective function for the approximate assignment of all the system eigenvalues:

$$J_2 = \sum_{i=1}^{n} L_i[r(\lambda_{di})]^2 + \sum_{j=1}^{m} M_j\ k_j^2 \tag{2.73}$$

where L_i represents the weighting on the i^{th} eigenvalue and M_j the weighting on the j^{th} element of $\underline{k}^T$. Further information on a modification of this approach will be presented in Chapter Four. A similar approach has been suggested in [19, 20] and its application in a simulation study involving the control of a gas-turbine has been very briefly described in [52]. These methods have the disadvantage of not differentiating between a positive and negative deviation of an eigenvalue from its desired value which is important in the stabilization of continuous-time systems. This difficulty is due to the choice of the objective functional in the minimization problem. An advantage of the method is that it results in a rather simple expression for $\underline{k}^T$. The obvious disadvantage in its application is the lack of information about conditions which ensure satisfactory eigenvalue assignment and a satisfactory transient and steady state response of the system.

Koenigsberg and Frederick's [49] and McBrinn and Roy's [53] approximate eigenvalue assignment techniques are gradient-search techniques based on the sensitivity of the system eigenvalues on the elements of the feedback gain matrix. Both papers [49, 53] attempt to shift the "poorly" located open-loop eigenvalues one-by-one to better positions without shifting the others significantly. The iterative procedure can be repeated as many times as desired although continual improvement of the results is not guaranteed. The sensitivity expressions used are due to Rosenbrock [54] and Reddy [55], who in turn have used the work of Fadeev and Fadeeva [56].

Still another eigenvalue sensitivity based method is that of McSparren and Etzweiler [57], whose method involves the change of some elements in the $\underline{A}$ matrix in order to approximately assign all of the system eigenvalues. They also suggest an iterative approach and provide some guidelines in the choice of state variables to be measured. A severe disadvantage of the method is that it is limited to only single-input systems which are completely controllable and observable.

The successful application of these methods to the stabilisation of up to 23rd order systems have been reported [49, 53, 57], which is encouraging.

Porter [58-60] has proposed the application of Liapunov's direct method in shifting the open-loop eigenvalues of a system to more stable locations thus stabilizing (or increasing the degree of stability) of a system. As noted by Johnson [61], Porter's method has extremely restricted applicability since it requires the existence of n independent measurements and controls, where n is the order of the system under consideration.

2.10 Use of Observers and Compensators in Eigenvalue Assignment

In systems where all of the state variables cannot be directly measured and the use of output feedback controllers does not provide adequate eigenvalue assignment, the designer has the option of using observers and compensators. This will ensure exact assignment of all of the system eigenvalues provided that the system is completely controllable and observable.

Reconstruction of all of the state variables by a reduced order Luenberger observer [62] and their subsequent use in a state feedback law ensures the assignment of the (2n-r) eigenvalues of the resulting closed-loop system. The separation property of the observers [62] allows the designer to separately specify the eigenvalues of the controlled system and the observer, and to design the observer and controller independently.

The application of this approach to physical systems has been reported in the literature [63, 64]. Sturgeon and Loscutoff [63] have tried to control a double inverted pendulum both in simulation and experimental studies. Their model of this system is 6th order with one control and three measurements, and their design approach consists of positioning the observer eigenvalues by the methods of [40] and those of the controlled system by the methods of [1, 5, 30]. Although simulation studies performed on an analog computer gave fairly good results, the experimental studies resulted in limit cycles [63]. Sturgeon and Loscutoff attribute this to modelling errors, the non-linearity of the real system and noise in the measurements. Their simulation studies clearly indicate that the observer rather than the controller is the source of the difficulties encountered. McMorran [64] in praising the Inverse Nyquist method [8] refers to Munro's Ph. D. thesis where an observer in conjunction with an eigenvalue assignment technique has been used. He blames the controller for the extremely poor response of the controlled system during regulatory operation. Simulation and experimental observer studies at the University of

Alberta [65] gave similar results for an optimal controller. The poor responses were not due to the controller but to the observer which can be quite sensitive to unmeasured disturbances.

The degree of the total system consisting of the controller and observer can be reduced if only some of the system eigenvalues have to be shifted. This can be achieved by the use of functional observers, which aim at reconstructing $\underline{\underline{C}}\ \underline{x}$ rather than $\underline{x}$. This method has been studied in [66, 67], but not applied to an eigenvalue assignment problem. Although this reduction of the observer order is important because of the high order of chemical systems, it still does not solve the vital problems associated with unseen disturbances. Smith and Davison [68] have suggested the obervation of disturbances. This approach may be promising, but requires further development.

The use of dual observers [62] is still another alternative for assigning the eigenvalues of the controlled system and the observer. In fact, the dual oberver design procedure devised by Murdoch [69] reduces the order of the observer to (ν_c-1), where ν_c is the controllability index of the controlled sytem.

Compensators

Pearson and his co-workers [70-71] have suggested augmenting the dynamic system to be controlled by an ingenuously designed dynamic system such that the total system has the desired set of eigenvalues. The dynamical system to be added, i.e., the dynamic compensator, has the minimum degree of $\min(\nu_c-1, \nu_o-1)$, where ν_c and ν_o are the controllability and observability indices of the system to be controlled. The compensator design technique suggested in [70, 71] is a time domain method and provides for the assignment of all of the eigenvalues of the augmented system. Thus, one does not have the option of specifying the eigenvalues of the controlled system and the compensator separately. Also, the designer cannot reduce the order of the compensator by requiring the assignment of only a subset of the system eigenvalues.

Ahmari and Vacroux [72] have suggested a compensator design technique based on that of [70, 71] and the eigenvalue assignment technique of [31, 35]. They have shown that for every compensator order selected there exists a unique maximum number of assignable eigenvalues. Shaw [73], on the other hand, has noted that for the assignment of certain sets of eigenvalues of the total system, the compensator might have to employ unstable eigenvalues. This, in turn, drastically increases the sensitivity of the controlled system eigenvalues to parameter variations in the system [73].

Chen [74] and Chen and Hsu [75] have proposed frequency domain compensators, whose minimal order is the same as in [70, 71]. The technique of [74, 75] allows the designer some freedom in configureating the compensator with the controlled system, Chen [74, 75] has considered two configurations, one of which allows independent specification of compensator and controlled system poles thus alleviating some of the disadvantages of the method of Pearson and his co-workers. But this extension may increase the order of the compensator.

The methods of this section have apparently not been applied to physical systems. They possess some design parameters which could be employed for design objectives in addition to eigenvalue assignment. The introduction of additional dynamics to already high order systems is rather difficult to justify unless the added dynamics can be purposfully employed.

A chief advantage of eigenvalue assignment techniques is that they are relatively easy to design and to implement. The use of observers and compensators in eigenvalue assignment tends to reduce this advantage considerably.

An interesting interpretation of the methods described in this section can be found in [76].

CHAPTER THREE

A NEW ALGORITHM FOR EIGENVALUE ASSIGNMENT USING OUTPUT FEEDBACK

3.1 Introduction

In recent years the design of control systems which provide for arbitrary pole assignment using output feedback control has received considerable attention [1, 2, 6-9]. Several design methods have been reported which employ a control matrix which is a dyadic product of two vectors [1, 2, 8]. Typically, the number of eigenvalues which can be arbitrarily assigned is restricted to max (m,r) where m and r are the dimensions of the output and control vectors, respectively [2]. In this chapter, a new algorithm is presented which allows min [m+r-1,n] poles to be assigned subject to certain mild restrictions. The algorithm provides a simple, analytical expression for the output control matrix as the sum of two dyadic products.

3.2 Formulation

Consider the completely controllable and observable system described by:

$$\left.\begin{aligned} \dot{\underline{x}} &= \underset{=}{A}\,\underline{x} + \underset{=}{B}\,\underline{u} \\ \underline{y} &= \underset{=}{C}\,\underline{x} \end{aligned}\right\} \qquad (3.1)$$

where $\underline{x}$ is an $n \times 1$ state vector, $\underline{u}$ is an $r \times 1$ control vector, $\underline{y}$ is an $m \times 1$ output vector, and $\underset{=}{A}$, $\underset{=}{B}$ and $\underset{=}{C}$ are constant matrices. Davison and Chatterjee [2] have given sufficient conditions for the arbitrarily close assignment of max (m,r) eigenvalues by the following linear, constant feedback law:

$$\underline{u} = \underline{\underline{G}}\,\underline{y} \; . \tag{3.2}$$

A relation between the closed-loop and open-loop characteristic polynomials, $r(\lambda)$ and $q(\lambda)$, of the closed-loop system consisting of (3.1) and (3.2) has been derived by Hsu and Chen [3]:

$$\frac{r(\lambda)}{q(\lambda)} = \det\,[\underline{\underline{I}} - \underline{\underline{C}}(\lambda\underline{\underline{I}} - \underline{\underline{A}})^{-1}\,\underline{\underline{B}}\,\underline{\underline{G}}] \; . \tag{3.3}$$

Assume that the feedback control matrix, $\underline{\underline{G}}$, is of the dyadic form

$$\underline{\underline{G}} = \underline{g}\,\underline{k}^T \; . \tag{3.4}$$

The resolvent matrix, $(\lambda\underline{\underline{I}} - \underline{\underline{A}})^{-1}$, for any $\lambda \neq 0$ can be expressed as [4]:

$$(\lambda\underline{\underline{I}} - \underline{\underline{A}})^{-1} = \frac{\underline{\underline{F}}(\lambda)}{q(\lambda)} = \frac{\underline{\underline{I}}\lambda^{n-1} + \underline{\underline{F}}_1\lambda^{n-2} + \ldots + \underline{\underline{F}}_{n-1}}{q(\lambda)} \tag{3.5}$$

where $\underline{\underline{F}}_1, \underline{\underline{F}}_2, \ldots, \underline{\underline{F}}_{n-1}$ can be obtained from Leverrier's algorithm [5]. Combining (3.3)-(3.5) gives [1]:

$$r(\lambda) = q(\lambda) - \underline{k}^T\,\underline{\underline{C}}\,\underline{\underline{F}}(\lambda)\,\underline{\underline{B}}\,\underline{g} \; . \tag{3.6}$$

The choice of a dyadic feedback matrix can be used to form two systems which are equivalent to the original system in (3.1) (3.2) and (3.4):

$$\text{System 1:} \quad \left.\begin{aligned} \dot{\underline{x}} &= \underline{\underline{A}}\,\underline{x} + \underline{b}\,u \\ \underline{y} &= \underline{\underline{C}}\,\underline{x} \end{aligned}\right\} \tag{3.7}$$

with

$$\underline{b} = \underline{\underline{B}}\,\underline{g} \tag{3.8}$$

$$u = \underline{k}^T\,\underline{y} \; . \tag{3.9}$$

System 2:
$$\left.\begin{aligned} \dot{\underline{x}} &= \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u} \\ y &= \underline{c}^T\,\underline{x} \end{aligned}\right\} \qquad (3.10)$$

with
$$\underline{c}^T = \underline{k}^T\,\underline{\underline{C}} \qquad (3.11)$$

$$\underline{u} = \underline{g}\,y \;. \qquad (3.12)$$

It is obvious that Systems 1 and 2 have the same closed-loop eigenvalues as the original system. Also, Ding, Brasch and Pearson [6], and Davison and Wang [7] have proved the existence of $\underline{g}$ and $\underline{k}^T$ which results in completely controllable and observable systems, respectively.

For System 1 algebraic manipulation of (3.6) gives

$$r(\lambda) = q(\lambda) - \underline{k}^T\,\underline{\underline{C}}\,\underline{\underline{Q}}\,\underline{\underline{U}}\,\underline{e} \qquad (3.13)$$

and for System 2:

$$r(\lambda) = q(\lambda) - \underline{g}^T\,\underline{\underline{B}}^T\,\underline{\underline{S}}\,\underline{\underline{U}}\,\underline{e} \;, \qquad (3.14)$$

where
$$\underline{\underline{Q}} = [\underline{b},\ \underline{\underline{A}}\,\underline{b},\ \underline{\underline{A}}^2\,\underline{b},\ \ldots,\ \underline{\underline{A}}^{n-1}\,\underline{b}] \qquad (3.15)$$

$$\underline{\underline{S}} = [\underline{c},\ \underline{\underline{A}}^T\,\underline{c},\ (\underline{\underline{A}}^T)^2\underline{c},\ \ldots,\ (\underline{\underline{A}}^T)^{n-1}\underline{c}] \qquad (3.16)$$

$$\underline{\underline{U}} = \begin{bmatrix} 1 & a_1 & a_2 & a_3 & \cdots & a_{n-1} \\ 0 & 1 & a_1 & a_2 & \cdots & a_{n-2} \\ 0 & 0 & 1 & a_1 & \cdots & a_{n-3} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \qquad (3.17)$$

$$\underline{e} = [\lambda^{n-1},\ \lambda^{n-2},\ \lambda^{n-3},\ \ldots,\ 1]^T \qquad (3.18)$$

and $a_1, a_2, \ldots, a_{n-1}$ are the coefficients of $q(\lambda)$, which can be determined from Leverrier's algorithm [5]. Thus (3.13) and (3.14) give the relations between the open-loop and closed-loop eigenvalues and the controllability and observability matrices, $\underline{Q}$ and $\underline{S}$, of Systems 1 and 2.

Davison and Wang [7] have shown that almost any output feedback control matrix will generate a closed-loop system with a set of eigenvalues disjoint from that of any corresponding completely controllable and observable open-loop system. Thus, in what follows, it will be assumed that any open-loop system (or equivalent system) either possesses distinct nonzero eigenvalues or has been converted into one with distinct nonzero eigenvalues.

3.3 Algorithm

In the derivation of the eigenvalue assignment algorithm, the two cases of $m \geq r$, and $r > m$ will be considered separately. In both cases the control law is expressed as

$$\underline{u} = \underline{u}_1 + \underline{u}_2 \tag{3.19}$$

where

$$\underline{u}_1 = \underline{g}_1 \underline{k}_1^T \underline{y} \tag{3.20}$$

$$\underline{u}_2 = \underline{g}_2 \underline{k}_2^T \underline{y} \quad . \tag{3.21}$$

Case 1: $m \geq r$

The algorithm involves three steps: assignment of m distinct eigenvalues, "protection" of $(r-1)$ of the eigenvalues (i.e., making them invariant to subsequent feedback control), and assignment of m additional distinct eigenvalues. Thus the total number of distinct eigenvalues assigned will be $m+r-1$ (or n if $m+r-1 > n$).

Step 1 (Assignment of m eigenvalues)

Choose a $\underline{g}_1$ vector such that the resulting equivalent system of (3.7) becomes:

$$\left.\begin{aligned} \dot{\underline{x}} &= \underline{\underline{A}}_1\,\underline{x} + \underline{b}_1\,u_1 \\ \underline{y} &= \underline{\underline{C}}\,\underline{x} \end{aligned}\right\} \tag{3.22}$$

where $\underline{\underline{A}}_1 = \underline{\underline{A}}$, $\underline{b}_1 = \underline{\underline{B}}\,\underline{g}_1$ and $u_1 = \underline{k}_1^T\,\underline{y}$. Evaluate (3.13) at m desired eigenvalues, $\lambda_{d1}, \lambda_{d2}, \ldots, \lambda_{dm}$, and impose the conditions that

$$r_1(\lambda_{d1}) = r_1(\lambda_{d2}) = \ldots = r_1(\lambda_{dm}) = 0 \tag{3.23}$$

where r_1 is the closed-loop characteristic polynomial for the system in (3.22). Then $\underline{k}_1$ can be determined from (3.24) below which is similar to Davison's algorithm [8]:

$$\underline{k}_1^T = \underline{q}_1\,(\underline{\underline{C}}\,\underline{\underline{Q}}_1\,\underline{\underline{U}}_1\,\underline{\underline{E}}_1)^{-1} \tag{3.24}$$

where

$$\underline{\underline{E}}_1 = \begin{bmatrix} \lambda_{d1}^{n-1} & \lambda_{d2}^{n-1} & \cdots & \lambda_{dm}^{n-1} \\ \lambda_{d1}^{n-2} & \lambda_{d2}^{n-2} & \cdots & \lambda_{dm}^{n-2} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 1 & 1 & 1 & 1 \end{bmatrix} \tag{3.25}$$

$$\underline{q}_1 = [q(\lambda_{d1}), q(\lambda_{d2}), \ldots, q(\lambda_{dm})] \tag{3.26}$$

and where $\underline{\underline{U}}_1 = \underline{\underline{U}}$ and $\underline{\underline{Q}}_1$ is obtained by replacing $\underline{b}$ with $\underline{b}_1$ in (3.15). Thus, as long as $(\underline{\underline{C}}\,\underline{\underline{Q}}_1\,\underline{\underline{U}}_1\,\underline{\underline{E}}_1)$ is nonsingular, (3.24) insures that $\lambda_{d1}, \lambda_{d2}, \ldots, \lambda_{dm}$ belong to the set of eigenvalues of the closed-loop system matrix $(\underline{\underline{A}}_1 + \underline{\underline{B}}\,\underline{g}_1\,\underline{k}_1^T\,\underline{\underline{C}})$. The invertibility of $(\underline{\underline{C}}\,\underline{\underline{Q}}_1\,\underline{\underline{U}}_1\,\underline{\underline{E}}_1)$ is determined by the choice of the desired eigenvalues.

Step 2 (Protection of (r-1) eigenvalues)

Consider the closed-loop system resulting from Step 1 as a new open-loop system given by:

$$\left.\begin{aligned} \dot{\underline{x}} &= \underline{\underline{A}}_2\,\underline{x} + \underline{\underline{B}}\,\underline{u}_2 \\ \underline{y} &= \underline{\underline{C}}\,\underline{x} \ , \end{aligned}\right\} \qquad (3.27)$$

where $\underline{\underline{A}}_2 = \underline{\underline{A}}_1 + \underline{\underline{B}}\,\underline{g}_1\,\underline{k}_1^T\,\underline{\underline{C}}$.

Output feedback affects all of the sytem eigenvalues which correspond to both controllable and observable modes [7], while none of the eigenvalues belonging to either an uncontrollable or unobservable mode can be affected by any output feedback controller [1], [9]. Thus, a subset of (r-1) eigenvalues of (3.27) can be "protected" by introducing an equivalent single-input system:

$$\left.\begin{aligned} \dot{\underline{x}} &= \underline{\underline{A}}_2\,\underline{x} + \underline{b}_2\,u_2 \\ \underline{y} &= \underline{\underline{C}}\,\underline{x} \end{aligned}\right\} \qquad (3.28)$$

with $\underline{b}_2 = \underline{\underline{B}}\,\underline{g}_2$ and $u_2 = \underline{k}_2^T\,\underline{y}$, in which the modes corresponding to these (r-1) eigenvalues have been made uncontrollable by an appropriate choice of $\underline{g}_2$. However, in order to assign m additional eigenvalues in Step 3, it is also necessary for the system in (3.28) to contain at least m controllable and observable modes. (Observability of the sytem in (3.27) is not affected by going to (3.28).) These conditions can be achieved by choosing $\underline{g}_2$ such that

$$\underline{\underline{V}}_2^1 \, \underline{\underline{B}} \, \underline{g}_2 = \underline{0} \tag{3.29}$$

and
$$\underline{\underline{V}}_2^2 \, \underline{\underline{B}} \, \underline{g}_2 = \underline{h}_2 \tag{3.30}$$

where $\underline{h}_2$ is an arbitrary vector with at least m nonzero elements and $\underline{\underline{V}}_2^1$ contains as its rows, (r-1) of the left eigenvectors of $\underline{\underline{A}}_2$ corresponding to (r-1) of the m eigenvalues which were assigned in Step 1, while $\underline{\underline{V}}_2^2$ has as its (n-r+1) rows, the remaining left eigenvectors of $\underline{\underline{A}}_2$.

Equation (3.29) can always be satisfied, since it requires finding an r-vector, $\underline{g}_2$, which is orthogonal to (r-1) r-vectors. In fact, if rank $(\underline{\underline{V}}_2^1 \, \underline{\underline{B}}) < r-1$, then the vector, $\underline{g}_2$ which satisfies (3.29) is not unique. The condition in (3.30) requires that the choice of $\underline{g}_2$ must be nonorthogonal to at least m vectors, which are a subset of the rows of $\underline{\underline{V}}_2^2 \, \underline{\underline{B}}$. (The number of rows of $\underline{\underline{V}}_2^2 \, \underline{\underline{B}}$ which fulfill this condition is equal to the degree of the relative minimal polynomial of $\underline{b}_2$ for the matrix $\underline{\underline{A}}_2$ [10].)

Remark: In situations where a particular choice of $\underline{g}_2$ satisfies (3.29) but not (3.30), several alternatives are available. One could return to Step 1 and choose a different $\underline{g}_1$, which will, in general, change the (n-m) unassigned eigenvalues of $(\underline{\underline{A}}_1 + \underline{\underline{B}} \, \underline{g}_1 \, \underline{k}_1^T \underline{\underline{C}})$ and consequently result in a new $(\underline{\underline{V}}_2^2 \, \underline{\underline{B}})$ matrix, or one could perturb the desired eigenvalues slightly and repeat Step 1 hoping that the resulting system will allow (3.29) and (3.30) to be satisfied. Alternatively, one could add another feedback controller with feedback gain matrix

$$\underline{\underline{G}}^* = \underline{g}_2 \, \underline{k}^{*T} \tag{3.31}$$

to (3.27) thus affecting its (n-r+1) eigenvalues and all of the system eigenvectors.

Step 3 (Assignment of another m eigenvalues)

Consider the equivalent system resulting from Step 2 as a new open-loop system given by (3.28). Application of Step 1 to this system will yield

$$\underline{k}_2^T = \underline{q}_2 (\underline{C}\, \underline{Q}_2\, \underline{U}_2\, \underline{E}_2)^{-1} \tag{3.32}$$

where $\underline{Q}_2$, $\underline{U}_2$, $\underline{E}_2$ and $\underline{q}_2$ pertain to the system in (3.28) and are analogous to the quantities defined in (3.15), (3.17), (3.25) and (3.26), respectively. It follows that rank $(\underline{U}_2) = n$, rank $(\underline{E}_2) = m$ and $n >$ rank $(\underline{Q}_2) \geq m$. The last inequality results from the sufficient condition set for successful application of Step 2, and the fact that the rank of $\underline{Q}_2$ is equal to the number of controllable modes of the system in (3.28) [10]. Thus the condition

$$\text{rank } (\underline{C}\, \underline{Q}_2\, \underline{U}_2\, \underline{E}_2) = m \tag{3.33}$$

is sufficient for the successful application of both Steps 2 and 3.

Case 2: $m < r$

The algorithm in this case is the dual of that described in Case 1. It involves: assignment of r eigenvalues, protection of (m-1) of these, and assignment of r additional eigenvalues. Thus, the total number of eigenvalues assigned is (m+r-1) provided that $(m+r-1) \leq n$.

Here the eigenvalue assignment is based on the use of the equivalent system given in (3.10) and the determination of a $\underline{k}_2^T$ in

Step 2 which makes (m-1) of the r modes (whose eigenvalues have been assigned in Step 1) unobservable while leaving at least r of the (n-m+1) remaining modes observable. In analogy with Case 1 the final control law becomes:

$$\underline{u} = \underline{g}_1 \underline{k}_1^T + \underline{g}_2 \underline{k}_2^T \tag{3.34}$$

where

$$\underline{g}_1 = (\underline{\underline{E}}_1^T \underline{\underline{U}}_1^T \underline{\underline{S}}_1^T \underline{\underline{B}})^{-1} \underline{q}_1^T \tag{3.35}$$

$$\underline{g}_2 = (\underline{\underline{E}}_2^T \underline{\underline{U}}_2^T \underline{\underline{S}}_2^T \underline{\underline{B}})^{-1} \underline{q}_2^T \tag{3.36}$$

and $\underline{\underline{S}}_1$, $\underline{\underline{S}}_2$, $\underline{\underline{U}}_1$ and $\underline{\underline{U}}_2$ are defined in analogy with (3.16) and (3.17) at each stage. $\underline{k}_2^T$ must fulfill the following conditions:

$$\underline{k}_2^T \underline{\underline{C}} \underline{\underline{W}}_2^1 = \underline{0} \tag{3.37}$$

$$\underline{k}_2^T \underline{\underline{C}} \underline{\underline{W}}_2^2 = \underline{f}_2 \tag{3.38}$$

where the m-vector $\underline{f}_2$ contains at least r nonzero elements. $\underline{\underline{W}}_2^1$ has as its columns (m-1) of the right eigenvectors of $\underline{\underline{\Lambda}}_2$ and $\underline{\underline{W}}_2^2$ has as its columns the remaining (n-m+1) right eigenvectors.

Cases 1 and 2 then form a constructive proof for the following proposition:

Proposition

If the system in (3.1) is completely controllable and observable with rank $\underline{\underline{C}} = m$ and rank $\underline{\underline{B}} = r$, then there exists a constant output feedback matrix $\underline{\underline{G}}$ such that min (m+r-1,n) eigenvalues of the closed-loop system can be assigned to arbitrary, distinct locations (with complex values chosen as conjugate pairs) provided that

$$\text{rank}\ (\underline{C}\ \underline{Q}_1\ \underline{U}_1\ \underline{E}_1) = \text{rank}\ (\underline{C}\ \underline{Q}_2\ \underline{U}_2\ \underline{E}_2) = m \quad \text{for} \quad m \geq r \qquad (3.39)$$

$$\text{rank}\ (\underline{E}_1^T\ \underline{U}_1^T\ \underline{S}_1^T\ \underline{B}) = \text{rank}\ (\underline{E}_2^T\ \underline{U}_2^T\ \underline{S}_2^T\ \underline{B}) = r \quad \text{for} \quad m < r\ . \qquad (3.40)$$

3.4 Example

Consider the following third order system with two control variables and two output variables:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -3 & 2 & 0 \\ 4 & -5 & 1 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad (3.41)$$

Its eigenvalues are given by

$$\lambda_1 = -1 \quad \lambda_2 = -3 \quad \lambda_3 - 7 \qquad (3.42)$$

and it is desired to find a control law, $\underline{u} = \underline{g}_1\ \underline{k}_1^T + \underline{g}_2\ \underline{k}_2^T$, such that the closed-loop eigenvalues become:

$$\lambda_{d1} = -10 \quad \lambda_{d2} = -9 \quad \lambda_{d3} = -8\ . \qquad (3.43)$$

Choose

$$\underline{g}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad (3.44)$$

then $\underline{k}_1^T$ can be determined from (3.24) as

$$\underline{k}_1^T = [-8.27 \quad 0.09] \quad . \tag{3.45}$$

The resulting system equation, $\underline{\underline{A}}_2 = \underline{\underline{A}}_1 + \underline{\underline{B}}\,\underline{g}_1\,\underline{k}_1^T\,\underline{\underline{C}}$, has as its eigenvalues:

$$\lambda_{d1} = -10.00 \qquad \lambda_{d2} = -5.00 \qquad \lambda_{d3} = -4.18 \quad .$$

Now, choose a $\underline{g}_2$ which satisfies (3.29) and (3.30) as

$$\underline{g}_2 = \begin{bmatrix} 2.571 \\ 1.000 \end{bmatrix} \tag{3.46}$$

$\underline{k}_2^T$ can be calculated from (3.32) as

$$\underline{k}_2^T = [-96.73 \quad 34.58] \tag{3.47}$$

which yields the final closed-loop system matrix, $\underline{\underline{A}}_3 = \underline{\underline{A}}_2 + \underline{\underline{B}}\,\underline{g}_2\,\underline{k}_2^T\,\underline{\underline{C}}$, whose eigenvalues are

$$\lambda_{d1} = -10.00 \qquad \lambda_{d2} = -9.00 \qquad \lambda_{d3} = -8.00 \quad . \tag{3.48}$$

It is worth noting that if previously reported design procedures [1, 2, 8] were employed, only $\max(m,r) = 2$ poles could be arbitarily assigned.

3.5 Conclusions

A new algorithm has been presented to achieve pole assignment using a proportional output feedback control law. Provided that certain mild conditions are satisfied, the number of poles that can be assigned to arbitrary, distinct locations is $\min[m+r-1,n]$ where m, r and n are the dimensions of the output, control and state vectors, respectively.

This number is a significant improvement over the max (m,r) poles which can be assigned using previously reported algorithms [1, 2, 8].

The algorithm consists of a two stage design procedure where some of the closed-loop eigenvalues which are shifted during the first stage are then "protected" by transforming the system into an equivalent single input (or single output) system. The equivalent system is chosen so that it satisfies the conditions of "ideal control" (or "ideal measurement") of Takahashi et al [12] since some of the modes have been made uncontrollable (or unobservable). Application of the second design stage allows assignment of additional poles without affecting the "protected" poles.

A numerical example demonstrates that the algorithm is quite simple and results in an analytical expression for the output control matrix.

CHAPTER FOUR

SIMULATION RESULTS FOR A DOUBLE EFFECT EVAPORATOR

4.1 Introduction

Some of the modal control methods described in Chapter Two and the eigenvalue assignment algorithm presented in Chapter Three have been applied to the fifth and third order state space models of the pilot scale, double effect evaporator in the Department of Chemical Engineering at the University of Alberta.

In this chapter the author has attempted to describe the design options available in the various design methods and to demonstrate their usefulness in fulfilling two design objectives in addition to specification of the closed-loop eigenvalues. These design objectives have been minimization of offset in the most important process variable, the product concentration, and some control over the magnitude of the gain elements in the controller matrices. These objectives have been deemed important, since integral action was not incorporated into the controllers used and excessively high gains would lead to difficulties in the future experimental evaluation of the controllers designed in this chapter.

The simplicity of the resulting controllers and the modest amounts of computer time required to design them have enabled the performance of a great number of controllers to be evaluated; only a few of these will be described here.

4.2 Physical Description and Mathematical Model of the Evaporator

Some of the control algorithms described and developed in Chapters Two and Three have been applied to the pilot plant scale, double effect evaporator schematically represented in Figure 4.1.

The first effect is a natural circulation calendria type unit which is fed with 5 lb./min. of 3.2 percent by weight of triethylene glycol. The feed is heated by 2 lb./min. of fresh steam. The second effect is an externally forced - circulation long tube vertical unit, which is heated by the first effect vapor. It produces about 1.5 lb./min. of ten percent triethylene glycol. The second effect vapor is totally condensed. Tight pressure control maintains the necessary pressure differential between the effects.

The systematic modeling of the evaporator under consideration and of similar units has been extensively studied at the University of Alberta [1-4]. Among the models developed for the double effect evaporator, a tenth order continuous-time nonlinear model derived from material and energy balances has been found to represent the experimental open-loop system behavior best [3].

The application of state space control methods devised for linear systems has necessitated the linearization of the nonlinear model. Open-loop experimental studies have indicated that this linearization introduces a significant modeling error unless the process variables stay roughly within a ten percent range of the steady state values around which linearization has been performed [3]. Regulatory control aims at holding the process variables close to their steady state values; thus, modeling errors introduced through linearization are expected to be smaller for the closed-loop system

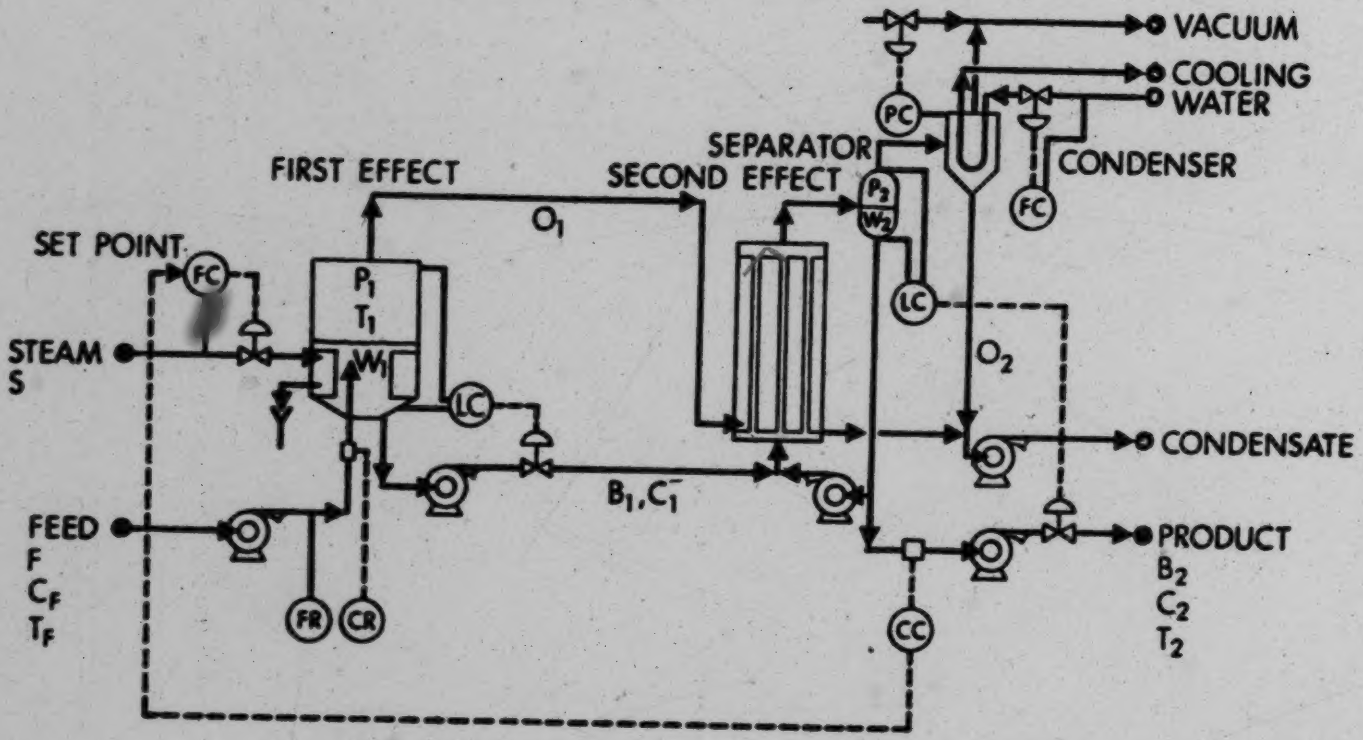

Figure 4.1 Schematic Diagram of Double Effect Evaporator

than for the open-loop one.

Both the design and implementation of multivariable control techniques can be carried out more easily and more economically with lower order models provided that these models predict the system behavior satisfactorily. The tenth order linearized evaporator model has been reduced to fifth and third order models through physical and modal considerations by Newell and Wilson [2-4]. Open-loop simulation and experimental studies have suggested that the use of reduced order models for multivariable control studies rather than the tenth order linear model is justifiable. This is due to the fact that the neglected modes of the higher order model are considerably faster than the retained ones.

The fifth and third order discrete-time evaporator models used in this chapter have been derived by Wilson [4]. They are of the form:

$$\underline{x}[(n+1)T] = \underline{\underline{\Phi}}\ \underline{x}(nT) + \underline{\underline{\Delta}}\ \underline{u}(nT) + \underline{\underline{\Theta}}\ \underline{d}(nT) \tag{4.1}$$

$$\underline{y}(nT) = \underline{\underline{C}}\ \underline{x}(nT) \quad , \tag{4.2}$$

where T denotes the time base or discretization interval. Wilson has applied Marshall's model reduction technique [5] to the tenth order linearized continuous-time model to obtain the fifth and third order continuous-time models, which then have been discretized with a time base of $T = 64$ seconds. This discretization is desirable for digital simulation and direct digital control applications.

The $\underline{\underline{\Phi}}$, $\underline{\underline{\Delta}}$, $\underline{\underline{\Theta}}$ and $\underline{\underline{C}}$ matrices for the fifth and third order discrete evaporator models are presented in Tables 4.1 and 4.2,

TABLE 4.1

Fifth Order Discrete Evaporator Model (T = 64 sec.)

$$\underline{\underline{\Phi}} = \begin{bmatrix} 1. & -0.0008 & -0.0912 & 0 & 0 \\ 0 & 0.9223 & 0.0871 & 0 & 0 \\ 0 & -0.0042 & 0.4376 & 0 & 0 \\ 0 & -0.0009 & -0.1052 & 1. & 0.0001 \\ 0 & 0.0391 & 0.1048 & 0 & 0.9603 \end{bmatrix}$$

$$\underline{\underline{\Delta}} = \begin{bmatrix} -0.0119 & -0.0817 & 0 \\ 0.0116 & 0 & 0 \\ 0.0116 & 0 & 0 \\ -0.0138 & 0.0848 & -0.0406 \\ 0.0137 & -0.0432 & 0 \end{bmatrix} \qquad \underline{\underline{\Theta}} = \begin{bmatrix} 0.1182 & 0 & -0.0050 \\ -0.0351 & 0.0785 & 0.0049 \\ -0.0136 & -0.0002 & 0.0662 \\ 0.0012 & 0 & -0.0058 \\ -0.0019 & 0.0016 & 0.0058 \end{bmatrix}$$

$$\underline{\underline{C}} = \begin{bmatrix} 1. & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1. & 0 \\ 0 & 0 & 0 & 0 & 1. \end{bmatrix}$$

TABLE 4.2

Third Order Discrete Evaporator Model (T = 64 sec.)

$$\underline{\Phi} = \begin{bmatrix} 1.0000 & 0.0000 & 0.0000 \\ 0.0000 & 1.0000 & 0.0000 \\ 0 & 0 & 0.9600 \end{bmatrix}$$

$$\underline{\Delta} = \begin{bmatrix} -0.0325 & -0.0811 & 0 \\ -0.0377 & 0.0854 & -0.0406 \\ 0.0527 & -0.0441 & 0 \end{bmatrix} \qquad \underline{\Theta} = \begin{bmatrix} 0.1200 & 0 & -0.0135 \\ 0.0032 & 0 & -0.0156 \\ -0.0218 & 0.0398 & 0.0218 \end{bmatrix}$$

$$\underline{C} = \begin{bmatrix} 1. & 0 & 0 \\ 0 & 1. & 0 \\ 0 & 0 & 1. \end{bmatrix}$$

TABLE 4.3

Description of the Evaporator Variables

State Vector, $\underline{x}$		Normal Steady State Value
$\underline{x}^T = [W1, C1, H1, W2, C2]$		
W1	First effect holdup	45.5 lb.
C1	First effect concentration	4.59% glycol
H1	First effect enthalpy	189.2 BTU/lb.
W2	Second effect holdup	41.5 lb.
C2	Second effect concentration	10.11% glycol
Control Vector, $\underline{u}$		
$\underline{u}^T = [S, B1, B2]$		
S	Steam flow	2.0 lb./min.
B1	First effect bottoms flow	3.485 lb./min.
B2	Second effect bottoms flow	1.581 lb./min.
Disturbance Vector, $\underline{d}$		
$\underline{d}^T = [F, CF, HF]$		
F	Feed flow	5.0 lb./min.
CF	Feed concentration	3.2% glycol
HF	Feed enthalpy	156.9 BTU/lb.
Output Vector, $\underline{y}$		
$\underline{y}^T = [W1, W2, C2]$		

respectively. The state, output, control, and disturbance vectors are defined in Table 4.3 and are expressed in normalized perturbation form, i.e.,

$$W1' \equiv \frac{W1-W1ss}{W1ss} \quad (ss \equiv \text{steady state}) \quad , \tag{4.3}$$

Their normal steady state values are also presented in Table 4.3. These same physical variables are involved in the third order model except that only W1', W2' and C2' have been retained as the state variables.

The accumulated experience on the experimental behavior of the evaporator over several years suggests that its closed-loop behavior deteriorates when the gain elements in the feedback matrices are excessively large. This has been attributed to neglecting noise effects, nonlinearities and time delays. The recommended upper limit on the value of gain elements is 100.0.

4.3 Modal Analysis of the Discrete Fifth Order Evaporator Model

Some of the design methods discussed in Section 2.6 will be applied here to the fifth and third order evaporator models. The eigenproperties of these models are in Table 4.4. The arrangement of the matrix element is such that

$$\underline{V}^T \underline{\Phi} \underline{W} = \underline{\Lambda} \ . \tag{4.4}$$

Table 4.5 present the mode controllability and mode observability matrices for the fifth order model.

Consideration of Tables 4.4, 4.5 reveals that:

1) The open-loop system has two unstable modes (corresponding

TABLE 4.4

Eigenproperties of the Fifth Order Evaporator Model

$$\underline{\underline{\Lambda}} = \begin{bmatrix} 1. & 0 & 0 & 0 & 0 \\ 0 & 0.9603 & 0 & 0 & 0 \\ 0 & 0 & 1. & 0 & 0 \\ 0 & 0 & 0 & 0.9215 & 0 \\ 0 & 0 & 0 & 0 & 0.4385 \end{bmatrix}$$

$$\underline{\underline{W}} = \begin{bmatrix} 1. & 0.0000 & -0.7080 & 0.0000 & -0.1526 \\ 0 & 0.0000 & 0.0000 & 0.7123 & 0.1693 \\ 0 & 0.0000 & 0.0000 & -0.0062 & -0.9413 \\ 0 & -0.0033 & -0.7062 & 0.0009 & -0.1759 \\ 0 & 1.0000 & 0.0000 & -0.7018 & 0.1763 \end{bmatrix}$$

$$\underline{\underline{V}}^T = \begin{bmatrix} 0.9871 & -0.0014 & -0.1602 & 0 & 0 \\ 0. & 0.6804 & 0.2514 & 0.0000 & 0.6884 \\ 0.6866 & -0.0006 & -0.2393 & 0.6866 & 0.0023 \\ 0.0000 & -0.9842 & -0.1771 & 0 & 0 \\ 0.0000 & -0.0087 & -1.0000 & 0 & 0 \end{bmatrix}$$

TABLE 4.5

Mode Controllability and Mode Observability Matrices of the Fifth Order Evaporator Model

$$\underline{\underline{H}} = \begin{bmatrix} 0.0056 & -0.1665 & 0.0407 \\ 0.0824 & -0.0432 & 0.0000 \\ 0.0607 & -0.1198 & 0.0575 \\ 0.0559 & -0.0000 & 0.0000 \\ -0.1169 & -0.0000 & 0.0000 \end{bmatrix}$$

$$\underline{\underline{F}} = \begin{bmatrix} 1.0000 & 0.0000 & -0.7080 & -0.0000 & -0.1526 \\ 0 & -0.0033 & -0.7062 & 0.0009 & -0.1760 \\ 0 & 1.0000 & -0.0000 & -0.7018 & 0.1764 \end{bmatrix}$$

to eigenvalues of 1.0).

2) The unstable modes possess repeated eigenvalues with distinct eigenvectors.

3) All of the system modes are both controllable and observable.

4) The system $\underline{\Phi}$ matrices are almost diagonal.

5) The right and left eigenvectors of the system are fairly well distributed in the state space.

The instability of the open-loop evaporator is due to the integrating nature of the two liquid levels involved. This type of instability is rather common in chemical engineering systems, and thus of considerable interest. Any satisfactory control systems design must be able to stabilize these unstable modes.

The presence of a pair of repeated eigenvalues with distinct eigenvectors implies that a unity rank feedback matrix cannot stabilize the system. This follows from well-known controllability considerations. In order to shift both of the unstable eigenvalues, the designer must either increase the rank of the feedback matrix or add an "insignificant" feedback control loop [6] to generate a new open-loop system with distinct eigenvalues. Alternatively, one could perturb the elements of $\underline{\Phi}$ slightly so that $\underline{\Phi}$ has distinct eigenvalues.

The approximately diagonal nature of the system matrix may imply that the eigenvectors form almost orthogonal sets, i.e., the right eigenvectors are almost orthogonal to each other and so are the left eigenvectors. Consideration of Table 4.4 indicates that this is not quite true. The reason for this is the high numerical sensitivity of the eigenvectors to the elements of the system matrix.

Thus, by considering the system matrix alone one cannot decide on the relative influence of each mode on each physical variable. But, with the eigenvectors well distributed in the state space it is not difficult to see that:

1) W1 is most influenced by the first mode, i.e., the mode whose right eigenvector of the first colum of $\underline{W}$ in Table 4.4.

2) C2 is most influenced by the second mode.

3) W2 is most influenced by the third mode.

It is also apparent that one cannot control the third mode without affecting the first mode considerably. Fortunately, this and similar interactions among the system modes have not been so severe as to produce unstable modes in any of the simulation studies to be described in the subsequent sections.

Next consider the open-loop response of the system to separate twenty percent step changes in feedflow, F, feed concentration, CF, and feed temperature, TF shown in Figure 4.2. The unstable response of the two holdups, W1 and W2, is apparent. The disturbance caused in the product concentration, C2, the main variable of interest is also significant. Thus, for any control strategy to succeed the first, the second, and the third modes must be controlled. This is also the reason for keeping these modes in the third order model.

The small dimensions of the system under consideration makes the configuration of the controller quite simple. The three measurements to be taken are obvious, and so are the three physical variables to manipulate in order to control W1, W2 and C2 effectively.

The fact that the eigenvectors are well distributed in the

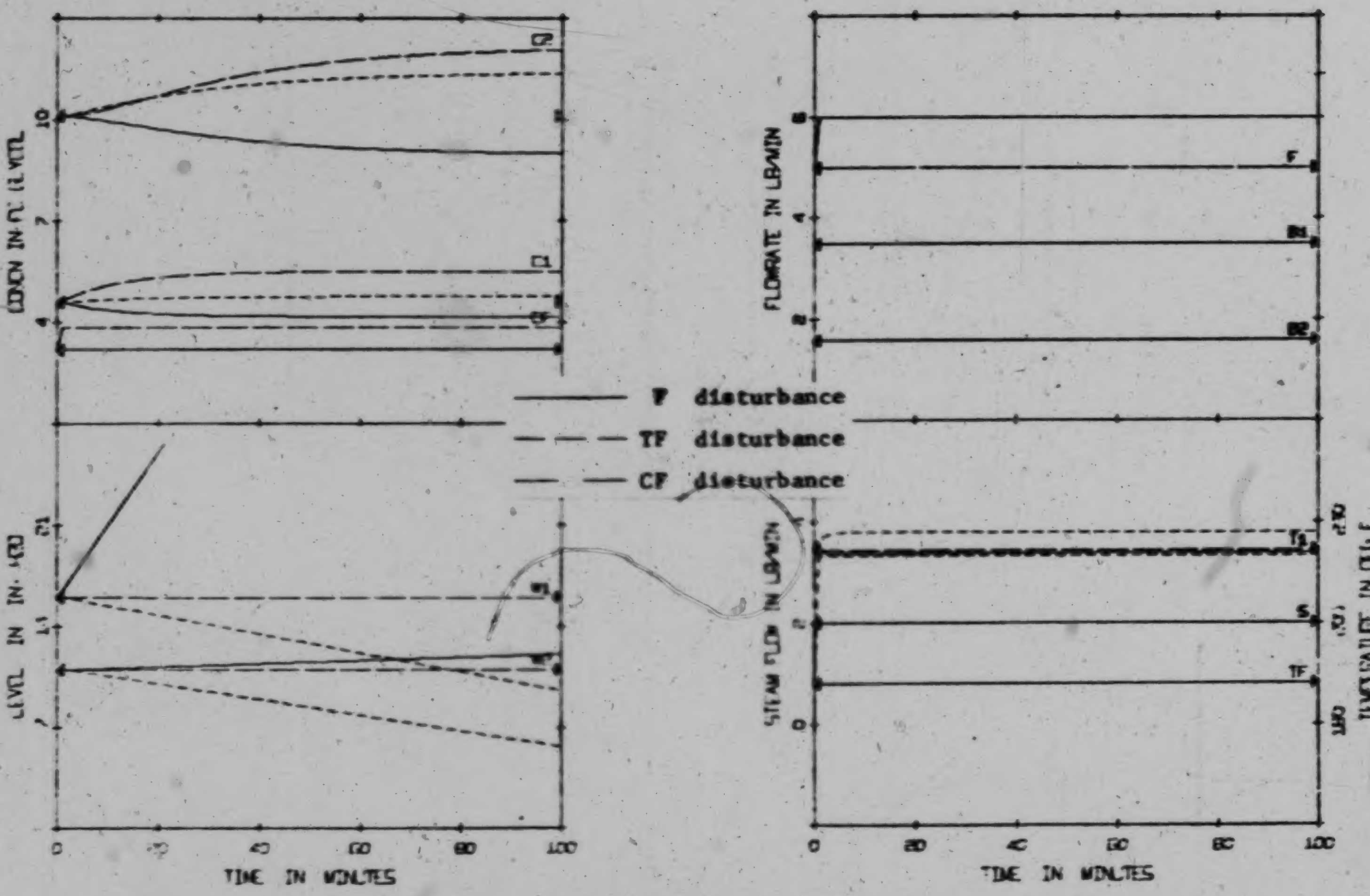

Figure 4.2 Open-loop Response (F, CF, TF Disturbances)

state space has two implications on the performance of the modal controllers to be designed:

1) The modes will not be highly interactive.

2) The magnitude of the controller gains required for a certain eigenvalue shift using modal control are expected not to be significantly greater than that required for any eigenvalue assignment technique.

The basis for the last argument can be found in McFarlane's interpretation of modal control [7].

4.4 Application of Rosenbrock's Approximate Modal Control Method [8] to the Fifth Order Evaporator Model

Rosenbrock's modal control technique has been simulated in this section. This technique has the desirable feature of providing some a priori indication of the success to be expected from its application to a specific physical system. This a priori evaluation for the evaporator model has been presented in Section 4.3. It is worth noting here again that the open-loop system eigenvectors are directionally well distributed in the state space and the eigenvalues corresponding to the dominant modes of the system are fairly well separated from the rest of the eigenvalues. In fact, the good distribution of the eigenvectors implies that each state variable is affected most by one particular mode which is different for each state variable.

The information about how well the eigenvalues of the controlled modes will be assigned can be obtained by applying Takahashi et al's analysis [9] described in Section 2.7. Application of their analysis [9] using the mode controllability matrix, $\underline{H}$, and the mode observability matrix, $\underline{F}$, given in Table 4.5 suggests that the approximate modal controllers to be designed can be expected to assign two of the system eigenvalues quite well. Also the eigenvalues belonging to the uncontrolled modes will not be moved significantly from their open-loop locations due to modal interactions. Specifically, this is indicated by the zero elements of the last two rows of the $\underline{H}$ matrix and the structure of the modal-domain closed-loop system matrix of (2.65). It should be noted though that the above arguments are only approximate, and unless either (2.62) or (2.63) holds, the exact assignment of any eigenvalue cannot be guaranteed.

The availability of three measurements and three controls implies that three modes can be approximately controlled [8, 9]. But, as indicated by the application of Takahashi et al's analysis [9] to the evaporator model, it will be possible to assign at most two of the three eigenvalues of the controlled modes.

After having decided on which modes to control as described in Section 4.4. The following design options are available and will be investigated in the simulation study:

1) The order in which the eigenvectors corresponding to the controlled modes appear in $\underline{W}_\ell$ and $\underline{v}_\ell^T$ which are used in the calculation of the controller matrix $\underline{G}$ via (2.61).

2) The specification of the desired eigenvalues, λ_{d1}, λ_{d2} and λ_{d3}.

3) Pairing of the open-loop eigenvalues with the desired closed-loop eigenvalues.

In the remainder of this section two major objectives of the discussion will be:

To demonstrate the usefulness of the above design options as design tools and to correlate the system closed-loop response, the eigenvalue locations, the eigenvector directions, and the characteristics of the controller matrices.

It should be noted that in cases where ideal modal controllers can be designed, only the second design option (above) has an effect on the resulting modal controller and thus on the performance of the resulting closed-loop system. But, in all other cases these design options, in general, can be used to obtain different controller matrices, thus different closed-loop eigenvalues, eigenvectors, and transient responses.

Simulation Study

In the simulation studies to be described in this and the following sections, disturbances in feed flow and feed concentration presented comparable challenges to the controllers, while disturbances in feed temperature were less severe. Thus, most of the simulation studies of this chapter will involve feed flow and feed concentration disturbances only. Also, the performance of all of the controllers will be evaluated by considering 20% step changes in feed flow and feed concentration beginning at time $t = 0$. This provides a common basis for the various comparisons that are made.

The eigenvalue and eigenvector calculations in this thesis have been performed on an IBM 360/67 computer. For this purpose the double precision version of the University of Alberta Computing Center subroutine CS201, based on Wilkinson's QR algorithm [10], has been used. The determination of the controller matrices and transient responses have been carried out on an IBM 1800 computer.

Table 4.8 reveals that the three last closed-loop eigenvalues resulting from all of the runs are fairly similar. Furthermore, inspection of Table 4.10 indicates that the product concentration C2, which is the most important controlled variable, is affected most by the two modes corresponding to the complex conjugate eigenvalues. This is one explanation for the small differences observed between the C2 responses resulting from the various runs.

The location of the closed-loop complex eigenvalues is fairly close to the origin of the unit circle in the z-plane. This accounts for the well-damped behavior of the transient responses observed in every run.

Effect of Eigenvector Order

The effect of the order in which the controlled mode eigenvectors appear in the eigenvector matrices is considered in Runs 1-6. The details of these studies are illustrated in Tables 4.6 - 4.10. Figures 4.3 and 4.4 compare the best and worse responses that were obtained. The difference in the closed-loop eigenvalue locations in Table 4.8 is admittedly not great, although the controller gains for Runs 1-6 differ significantly (cf. Table 4.9). Two of the eigenvalues are fairly well assigned, as would be expected from the earlier discussion.

The effect of the order in which the uncontrolled mode eigenvectors appear in the eigenvector matrices is investigated in Runs 7 and 8. The resulting closed-loop eigenvalues and controller matrices were identical. This is entirely expectable since the uncontrolled mode eigenvectors do not have any effect on the controller matrix designed.

Desired Eigenvalues

In choosing real desired closed-loop eigenvalues two points must be considered: the absolute and the relative magnitude of the eigenvalues. Runs 9-12 consider the first point. It is apparent from Figures 4.5 - 4.7 that the transient response improves with decreasing magnitude of the eigenvalues. Run 12 is not shown since its transient responses were essentially identical to those in Run 11. Consideration of Table 4.7 and Figures 4.5 - 4.7 indicates that this change is not directly proportional to the decrease in the magnitude of the desired eigenvalues. Also, comparison of Runs 11 and 12 clearly indicates that the amount of improvement to be expected by decreasing the value of the eigenvalues reaches a limit. Comparison of the corresponding closed-loop eigenvalues and eigenvectors indicates that there is only a small change in the eigenvector directions and also in the complex conjugate closed-loop eigenvalues which belong to the closed-loop modes most affecting C2. This suggests the importance of the closed-loop modes rather than just the closed-loop eigenvalues.

Similar observations can be made about the values of the gain elements in the corresponding controller matrices: they appear to tend toward limiting values as the desired eigenvalues become smaller. Another observation of some practical value, especially in the particular case study, is that the increase in the values of the gain elements has not caused any nonminimum phase behavior, which might have occurred according to MacFarlane [7]. Similarly, the largest gain elements are considerably smaller than the upper limit of 100.0 set in Section 4.2.

The second point, concerning the relative magnitude of the desired eigenvalues, has been illustrated in Runs 10, 13-15. Although the sum of the three desired eigenvalues is comparable the resulting controller matrices and the closed-loop responses in Figures 4.8 and 4.9 are significantly different. Comparison of Runs 7, 12 and 17 indicates that the relative magnitude of the eigenvalues has very small influence on the values of the gain elements in cases where all of the desired eigenvalues have small magnitudes. The runs considered in this paragraph demonstrates how important this design option can be in influencing the magnitude of the various gain elements, particularly in cases where some of the gain elements become undesirably large.

Pairing of Open-loop and Desired Closed-loop Eigenvalues

The last design option, pairing of the desired closed-loop eigenvalues with the open-loop eigenvalues, has been considered in Runs 15, 18 and 19. Run 19, where the smallest desired eigenvalue was paired with the eigenvalue of the mode influencing both W1 and W2, resulted in the best controller while the controllers in Runs 15 and 18 were quite similar as demonstrated in Figures 4.10 and 4.11. Thus this design option should not be ignored although it is not obvious a priori which pairing will result in the best combination.

A closer look at the controller matrices given in Table 4.10 suggests that all of them are quite similar. It is very difficult to interpret multivariable control laws by considering the individual gain elements in the controller matrices. But, extensive experience with the evaporator system and the multi-loop controllers designed in previous investigations [11, 12] indicates that each gain element in the controller matrix has a physically reasonable sign and magnitude with the exception of the small elements in the (1,2) and (2,2) positions which tend to change sign from run to run.

Since driving the modal activations to zero should eventually result in driving the state variables to their steady state values, the behavior of the manipulated variables will be physically meaningful as long as the modes are not highly interactive. This has beeen observed in all of the simulation studies of this section.

Summary

The following observations were made concerning the application of Rosenbrock's approximate modal control method [8] to the fifth order evaporator model:

1) The product concentration, C2, which is the variable of greatest interest, showed a small offset and was well-behaved. The other state variables were also satisfactorily controlled.

2) The manipulated variables, S, B1 and B2, showed in every run the physically expected behavior and never exceeded the physical limits of the actual evaporator.

3) Two of the system eigenvalues could be fairly well assigned. Two of the other eigenvalues degenerated into a complex conjugate pair which changed very little from run to run.

4) As the desired eigenvalues approached the limit zero the closed-loop eigenvectors approached limiting directions. Similarly, the gain elements in the controller matrices approached limiting values, which were reasonably small.

9) The closed-loop dominant modes were those pertaining to the complex conjugate eigenvalues. Since these eigenvalues could not be directly affected the corresponding responses were similar in every run as

noted in point 1.

10) The choice of desired closed-loop eigenvalue locations, ordering of the controlled mode eigenvectors in the eigenvector matrices, and the pairing of open-loop and closed-loop eigenvalues proved to be useful design options.

It is very difficult, if not impossible, to generalize the details of the evaporator results to other systems. But it is possible to draw some general conclusions:

1) In systems with well-distributed open-loop eigenvectors and well separated eigenvalues, Rosenbrock's approximate modal controller [8] is expected to give satisfactory results.

2) If one can also manage to design a control law with distinctly dominant closed-loop modes with desirable eigenvalues, the performance of the closed-loop system is expected to be very satisfactory unless the remaining closed-loop eigenvalues possess very undesirable locations.

4.5 Application of Takahashi et al's Approximate Modal Control Method [9] to the Fifth Order Evaporator Model

Takahashi et al's approximate modal control method [9], which has been described in Section 2.7, lacks a theoretical basis for predicting its suitability for controlling a specific system. Thus it seems rather pointless to attempt a detailed study for a specific sytem. Consequently, only representative results were sought in this investigation.

All of the design freedoms described and evaluated in Section 4.4 exist for Takahashi's method, too. But, here only the third design option has been evaluated. Experience with the various options in Section 4.4 has suggested that this option has the greatest influence on the response characteristics of the closed-loop system. The order in which the eigenvectors appear in the eigenvector matrices has been selected as the order used in Run 1.

Takahashi et al's method [9] allows the option of assigning all of the system eigenvalues. This was attempted in Runs 20 and 21. Table 4.13 indicates that although the eigenvalues have generally been shifted in the appropriate direction the eigenvalue assignement is very poor. The response characteristics in Figures 4.12 and 4.13 are also worse than any of the ones obtained in Section 4.4. In Runs 22 and 23 two of the desired closed-loop eigenvalues were specified to be the open-loop eigenvalues of 0.1921 and 0.438 (i.e., only three of the open-loop eigenvalues were to be changed).

Table 4.12 and Figures 4.12 and 4.13 clearly indicate that although five eigenvalues are shifted this strategy has significantly improved the response characteristics since the response times and offsets are reduced.

The results in Table 4.13 indicate that it is very difficult to see a trend in how well the eigenvalues have been assigned. Similarly, it has not been possible to observe a trend in the closed-loop eigenvectors, which are therefore not reported. Consideration of Table 4.14, on the other hand, is very informative: Runs 20-23, although performed for different specifications of the desired eigenvalues, have resulted in very similar controller matrices. This may have a mathematical explanation since these controller matrices represent the least squares solution of a set of linear algebraic equations.

Apparently these equations are not very sensitive to the desired eigenvalues used in Table 4.11. It is possible that use of the several design freedoms described in Section 4.4 can significantly affect the value of the gain elements and could help to improve the poorer response characteristics observed.

A closer look at the controller matrices pertaining to the first four runs of this section indicates that gain element (1,3) in $\underset{\sim}{G}$ relating product concentration, C2, and steam flow rate, S, is much smaller than those of the previous section. In Run 24 this gain element has been assigned a value of -10.00. The drastic improvement in the response characteristics (cf., Figures 4.12 and 4.13) indicates that intuition may be helpful, even in multivariable control.

4.6 Application of Model Reduction and Ideal Modal Control to the Fifth Order Evaporator Model

In Section 2.9 it was noted that model reduction offers an alternative to the application of approximate modal control techniques. Specifically, the designer can generate a reduced order system model with the same number of state variables as control and output variables which then can be used to design an ideal modal controller. The application of the ideal modal controller to the original model will form the real test for the justification of the approach used. Many model reduction techniques have been proposed in the literature. But, since the resulting model will be employed to design an ideal modal controller, Marshall's [5] model reduction technique based on the modal characteristics of the system has been deemed the most suitable.

Reduction of the fifth order evaporator model to the third order model described in Section 4.2 generates a system with 3 states, 3 outputs, and 3 controls. Thus, the application of the ideal modal control technique described in Section 2.3 is possible.

Two simulation studies were performed in which the desired eigenvalue sets were:

$$\{0.0000, \quad 0.0000, \quad 0.0000\} \tag{4.5}$$

and

$$\{0.0006, \quad 0.0005, \quad 0.0004\} \; . \tag{4.6}$$

The resulting third order closed-loop matrices were exactly assigned these eigenvalues and the closed-loop eigenvectors were identical to the open-loop ones. Similarly, the resulting controller matrices were:

$$\underline{G} = \begin{bmatrix} 7.729 & .0002 & -13.63 \\ 9.233 & -.0000 & 5.465 \\ 12.23 & 24.61 & 24.14 \end{bmatrix} \tag{4.7}$$

and

$$\underline{G} = \begin{bmatrix} 7.639 & -.0491 & -13.63 \\ 9.125 & -.0591 & 5.63 \\ 12.57 & 24.79 & 24.13 \end{bmatrix} \; . \tag{4.8}$$

At this point one might note the similarity of these controller matrices to each other and especially to those obtained in Section 4.4 for small eigenvalues.

Two tests were applied to evaluate the success of this approach:

1) Determination of the eigenvalues of the fifth order closed-

loop system matrix resulting from the use of the controller matrices in (4.7) and (4.8).

2) Determination of the closed-loop response of the fifth order model obtained by the controller matrices of (4.7) and (4.8).

The results of these tests were very satisfactory. The eigenvalues of the fifth order closed-loop system matrices were:

$$\{0.001, \quad 0.001, \quad 0.576 \pm 0.562i, \quad 0.898\} \qquad (4.9)$$

and

$$\{0.002, \quad -0.005, \quad 0.577 \pm 0.561i, \quad 0.898\} \; . \qquad (4.10)$$

They are by no means identical to the eigenvalues of the third order closed-loop system matrices, but they are certainly much better than the open-loop system eigenvalues. The closed-loop system response to F, CF, and TF disturbances was almost identical using these two controllers. As depicted by Figures 4.14, 4.15 and 4.16 the closed-loop responses obtained in this section are better than any of those obtained in Sections 4.4 and 4.5. The offsets resulting from the two studies of this section were -0.002, 0.007, and 0.004 for F, CF, and TF disturbances, respectively.

The success of this combination consisting of a modal approach to model reduction and ideal modal control can by no means ~~be~~ generalized to other applications. Specifically, one cannot claim that this approach is superior to the approximate modal control techniques of [8, 9]. But, it is possible to say that, whenever the open-loop system possesses well separated eigenvalues and well distributed eigenvectors, the chances of this approach providing satisfactory results are quite good.

4.7 Application of the New Eigenvalue Assignment Method to the Fifth Order Evaporator Model

In this section the eigenvalue assignment algorithm developed in Chapter Three has been applied to the fifth order evaporator model. This algorithm allows the designer to assign $(m+r-1)$ eigenvalues of a system provided that it fulfills the sufficient conditions of the proposition presented in Chapter Three. Since the evaporator model fulfills these conditions, it was possible to assign all of the five of its eigenvalues.

The various steps involved in the application of the method to the evaporator model have been described in Table 4.15. Vectors $\underline{g}_1$ and $\underline{g}_2$ refer to the column vectors involved in Step 1 and Step 2 of the algorithm and the asterisk superscripts designate those eigenvalues whose modes have been made uncontrollable in Step 2 of the algorithm, (i.e., these eigenvalues were assigned in Step 1 and did not change in Step 2).

Initial attempts to apply the algorithm to the evaporator model encountered the difficulty of generating high gains in the controller matrices as illustrated by Runs 25 and 27. The controller matrix $\underline{G}_{25}$ shown in Table 4.18 contains gain elements which are unacceptably large for the satisfactory control of the actual evaporator. As noted by Fallside and Seraji [13] the magnitude of the gain elements in the controller matrix depends on the ratio of the elements in the $\underline{g}$ vectors. Since the choice of $\underline{g}_1$ is arbitrary as long as the corresponding equivalent system remains controllable, this design freedom can be used to reduce the magnitude of the gain elements. The controller matrix obtained in Run 26 has resulted from an extensive trial and error search for such a g_1. Although most of the gain elements of Run 26 are considerably smaller than those of Run 25, one of them is still greater in absolute value than the limit of 100.00 set in Section 4.2.

Rather than assigning m of the sytem eigenvalues simultaaneously in Step 1, they can be assigned recursively, one or more at a time. The question then arises as to how g_1 should be chosen for each recursive part of Step 1. In dyadic state feedback methods, where pairing of closed-loop and open-loop eigenvalues is possible, Simon and Mitter's [14] strategy of choosing g so as to maximize the controllability of the single eigenvalue to be assigned can be employed (cf., Section 2.4). However, in output feedback techniques one does not have a priori knowledge about which open-loop eigenvalue will be shifted to the desired closed-loop location. Thus, a modification of Simon and Mitter's approach [14] is needed.

The unique strategy adopted in this thesis was to perform Step 1 of the algorithm developed in Chapter Three such that only one eigenvalue would be assigned recursively. Thus, rather than assigning three eigenvalues their desired closed-loop locations only one was assigned this value while the other two were assigned their respective open-loop locations. The three possible g_1's fulfilling equation (2.55) were then tested and the one giving the smallest gains was used in the final design. It should be noted though that in most of the runs only two recursive steps were performed in Step 1, since the resulting eigenvalue locations were deemed satisfactory.

Run 27 and Run 28 clearly indicate the success of this approach in reducing the magnitude of the controller gains. In fact, in all of the runs where this approach was used, none of the controller matrix elements exceeded a value of 45 for the desired eigenvalues under consideration. However smaller desired eigenvalues resulted in larger gains, as would be expected.

The major design options in this eigenvalue assignment method are:

1) Choice of the desired closed-loop eigenvalue locations.

2) Choice of the $\underline{g}$'s to be used.

3) Choice of the number of recursive substeps to be used in Step 1 of the algorithm.

The second design option has been utilized in order to ensure gain elements of reasonable magnitude in the controller matrices as described above.

Runs 29-31 illustrated the fact that both the choice of desired closed-loop eigenvalues and the number of recursive steps used affect the closed-loop transient response characteristics considerably. This is in agreement with the arguments put forward in Chapter Two: the response characteristics of a system are not only governed by its eigenvalues but also by its eigenvectors whose directions depend on the type of eigenvalue assignment scheme employed. It is obvious that the choice of the $\underline{g}$'s directly affects the closed-loop eigenvectors, but the analytical relation between these variables is not known, yet.

Runs 32-35 are presented to indicate that even comparatively small changes in one of the five closed-loop eigenvalues of the system can result in considerably different response characteristics as shown in Figures 4.19 and 4.20.

Runs 36 and 37 depicted in Figures 4.21 - 4.24 are the best results obtained in a large number of applications (> 100) of the new algorith to the evaporator model. The reason for their inclusion in this thesis is to convince the reader that eigenvalue assignment can give excellent results if one is prepared to make exhaustive use of the available design options.

Runs 38 and 39 use only the first part of Runs 36 and 37, respectively. Figures 4.25 and 4.26 clearly indicate the need of using a recursive scheme in this particular application where repeated eigenvalues with distinct eigenvectors occur.

The following observations are made concerning the application of the eigenvalue assignment technique developed in Chapter Three to the evaporator model:

1) Significant variations in the controller matrices and the closed-loop transient responses were observed.

2) CF disturbances were more severe than TF and F disturbances and generally caused significant offsets in the controlled variables. This was due to the fact that in every run the steam flowrate reached a steady state below its normal steady state value rather than above it. B1 and B2 were always properly manipulated.

3) By changing the value of a small set of desired eigenvalues, one could always reach a satisfactory closed-loop performance.

4) Use of a greater number of linearly independent $\underline{g}$'s in recursive steps in general, tended to equalize the magnitude of gain elements.

5) The large gain elements used in Run 29 caused some overshoot to the feedflow disturbance.

The simulation studies of this section clearly indicate that the objective of gaining some control over the entire set of open-loop eigenvalues is a more desirable one than the exact assignment of some of the eigenvalues. This point may be appreciated better in applications where the dimension of the system model is much larger than the number of available measurements and controls. In such instances the combination of exact and approximate eigenvalue assignment methods by the application of the following procedure may be useful:

1) Assign max (m,r) of the system eigenvalues by the application of Step 1 of the algorithm described in Chapter Three.

2) Determine $\underline{g}_2$ or $\underline{k}_2^T$ by the application of Step 2 of the same algorithm in order to protect max (m-1, r-1) of the already assigned eigenvalues.

3) Apply Jameson's [15] approximate eigenvalue assignment technique by employing $\underline{g}_2$ designed above. This approach will minimize the objective function defined in (2.73) and approximately assign those of the remaining max (n-m+1, n-r+1) system eigenvalues which belong to both controllable and observable modes.

4.8 Conclusions

In this section some of the more important observations made in the simulation studies will be cited. Their interpretation and correlation will be presented in Chapter Five.

Controllers derived via the application of Rosenbrock's approximate modal control method [8] produced closed-loop systems with similar response characteristics for any set of design options and disturbances considered. The location of the desired closed-loop eigenvalues was the most important design option. Specification of smaller eigenvalues uniformly improved the response characteristics of the system, but this improvement was not limitless.

Controllers derived via the application of the eigenvalue assigment method described in Chapter Three resulted in closed-loop systems with drastically different response characteristics for each set of design options and for different disturbances. The specification of the $\underline{g}$ vectors and the number of recursive steps employed greatly affected the response characteristics of the closed-loop system.

The gains in the controller matrices of the eigenvalue assignment technique were in general, higher than those of Rosenbrock's [8] method for the same set of desired closed-loop eigenvalues.

Sufficient patience in the use of the various design options available in each method allowed determination of controllers which provided good response characteristics. In fact, for the disturbances employed no serious need appeared for the addition of integral action. This may not hold though in experimental studies, where modeling errors and noise effects become important.

TABLE 4.6

Design Specifications for Rosenbrock's Approximate Modal Control Method [8]

Run Number	Ordering of Eigenvectors*					Desired Eigenvalues		
1	$\underline{w}_1$	$\underline{w}_2$	$\underline{w}_3$	$\underline{w}_4$	$\underline{w}_5$	0.3	0.5	0.7
2	$\underline{w}_2$	$\underline{w}_1$	$\underline{w}_3$	$\underline{w}_4$	$\underline{w}_5$	0.3	0.5	0.7
3	$\underline{w}_3$	$\underline{w}_2$	$\underline{w}_1$	$\underline{w}_4$	$\underline{w}_5$	0.3	0.5	0.7
4	$\underline{w}_1$	$\underline{w}_3$	$\underline{w}_2$	$\underline{w}_4$	$\underline{w}_5$	0.3	0.5	0.7
5	$\underline{w}_2$	$\underline{w}_3$	$\underline{w}_4$	$\underline{w}_4$	$\underline{w}_5$	0.3	0.5	0.7
6	$\underline{w}_3$	$\underline{w}_1$	$\underline{w}_2$	$\underline{w}_4$	$\underline{w}_5$	0.3	0.5	0.7
7	$\underline{w}_1$	$\underline{w}_2$	$\underline{w}_3$	$\underline{w}_4$	$\underline{w}_5$	0.00006	0.0003	0.001
8	$\underline{w}_1$	$\underline{w}_2$	$\underline{w}_3$	$\underline{w}_5$	$\underline{w}_4$	0.00006	0.0003	0.001
9	$\underline{w}_1$	$\underline{w}_2$	$\underline{w}_3$	$\underline{w}_4$	$\underline{w}_5$	0.8	0.8	0.8
10	$\underline{w}_1$	$\underline{w}_2$	$\underline{w}_3$	$\underline{w}_4$	$\underline{w}_5$	0.5	0.5	0.5
11	$\underline{w}_1$	$\underline{w}_2$	$\underline{w}_3$	$\underline{w}_4$	$\underline{w}_5$	0.05	0.05	0.05
12	$\underline{w}_1$	$\underline{w}_2$	$\underline{w}_3$	$\underline{w}_4$	$\underline{w}_5$	0.00005	0.00005	0.00005
13	$\underline{w}_1$	$\underline{w}_2$	$\underline{w}_3$	$\underline{w}_4$	$\underline{w}_5$	0.5	0.4	0.6
14	$\underline{w}_1$	$\underline{w}_2$	$\underline{w}_3$	$\underline{w}_4$	$\underline{w}_5$	0.1	0.4	0.7
15	$\underline{w}_1$	$\underline{w}_2$	$\underline{w}_3$	$\underline{w}_4$	$\underline{w}_5$	0.4	0.0001	0.7
16	$\underline{w}_1$	$\underline{w}_2$	$\underline{w}_3$	$\underline{w}_4$	$\underline{w}_5$	0.4	0.0001	0.0007
17	$\underline{w}_1$	$\underline{w}_2$	$\underline{w}_3$	$\underline{w}_4$	$\underline{w}_5$	0.0004	0.0001	0.0007
18	$\underline{w}_1$	$\underline{w}_2$	$\underline{w}_3$	$\underline{w}_4$	$\underline{w}_5$	0.0001	0.4	0.7
19	$\underline{w}_1$	$\underline{w}_2$	$\underline{w}_3$	$\underline{w}_4$	$\underline{w}_5$	0.4	0.7	0.0001

---

*The eigenvectors $\underline{w}_1, \ldots, \underline{w}_5$ are the corresponding columns of matrix $\underline{\underline{W}}$ in Table 4.4.

TABLE 4.7

Response Characteristics for Rosenbrock's Approximate Modal Control Method [8]

Run Number	Corresponding Figures	Offset in C2' for F Disturbance	Offset in C2' for CF Disturbance
1	4.3, 4.4	-0.013	0.014
2		-0.018	0.020
3		-0.018	0.020
4		-0.013	0.014
5		-0.030	0.033
6	4.3, 4.4	-0.030	0.033
7		-0.009	0.010
8		-0.009	0.010
9	4.5, 4.6, 4.7	-0.043	0.048
10	4.5, 4.6, 4.7, 4.8 and 4.9	-0.018	0.020
11	4.5, 4.6, 4.7	-0.009	0.010
12		-0.009	0.010
13	4.8, 4.9	-0.022	0.025
14		-0.030	0.033
15	4.10, 4.11	-0.030	0.033
16		-0.009	0.010
17		-0.009	0.010
18		-0.030	0.033
19	4.10, 4.11	-0.009	0.010

TABLE 4.8

Actual Closed-loop Eigenvalues for Rosenbrock's [8]
Approximate Modal Control Method

Run Number	Closed-loop Eigenvalues			
1	0.520	0.607	0.657±0.269i	0.897
2	0.387	0.625	0.697±0.201i	0.895
3	0.383	0.708	0.642±0.258i	0.894
4	0.282	0.771	0.580±0.148i	0.885
5	0.441	0.706	0.665±0.323i	0.896
6	0.527	0.776	0.472±0.171i	0.885
7	0.000	0.001	0.620±0.444i	0.898
8	0.000	0.001	0.620±0.444i	0.898
9	0.790	0.823	0.879±0.035i	0.538
10	0.486	0.500	0.672±0.242i	0.894
11	0.049	0.050	0.625±0.431i	0.897
12	0.000	0.000	0.620±0.447i	0.898
13	0.419	0.555	0.653±0.184i	0.892
14	0.079	0.774	0.535±0.161i	0.885
15	0.500	0.779	0.311±0.139i	0.884
16	0.000	0.412	0.618±0.450i	0.900
17	0.000	0.000	0.628±0.439i	0.881
18	0.023	0.774	0.536±0.162i	0.885
19	0.428	0.606	0.604±0.342i	0.898

TABLE 4.9

Controllers for Rosenbrock's Approximate Modal Control Method [8]

$$\underline{\underline{G}}_1 = \begin{bmatrix} 2.589 & -1.038 & -6.465 \\ 4.936 & -1.979 & 2.950 \\ 7.546 & 4.360 & 12.99 \end{bmatrix}$$

$$\underline{\underline{G}}_2 = \begin{bmatrix} 3.625 & -2.077 & -4.511 \\ 6.910 & -3.958 & 2.048 \\ 10.56 & 1.334 & 9.011 \end{bmatrix}$$

$$\underline{\underline{G}}_3 = \begin{bmatrix} 1.554 & 2.077 & -4.497 \\ 2.962 & 3.958 & 2.074 \\ 4.527 & 23.28 & 9.126 \end{bmatrix}$$

$$\underline{\underline{G}}_3 = \begin{bmatrix} 1.554 & 1.038 & -6.458 \\ 2.962 & 1.979 & 2.963 \\ 4.527 & 15.33 & 13.03 \end{bmatrix}$$

$$\underline{\underline{G}}_5 = \begin{bmatrix} 3.625 & -1.038 & -2.551 \\ 6.910 & -1.979 & 11.59 \\ 10.56 & 9.283 & 5.148 \end{bmatrix}$$

$$\underline{\underline{G}}_6 = \begin{bmatrix} 2.589 & 1.038 & -2.544 \\ 4.936 & 1.979 & 1.172 \\ 7.546 & 20.26 & 5.184 \end{bmatrix}$$

$$\underline{\underline{G}}_7 = \begin{bmatrix} 5.178 & -.0049 & -9.394 \\ 9.871 & -.0093 & 4.298 \\ 15.09 & 24.58 & 18.95 \end{bmatrix}$$

$$\underline{\underline{G}}_8 = \begin{bmatrix} 5.178 & -.0049 & -9.394 \\ 9.871 & -.0093 & 4.298 \\ 15.09 & 24.58 & 18.95 \end{bmatrix}$$

$$\underline{\underline{G}}_9 = \begin{bmatrix} 1.043 & -.0000 & -15.28 \\ 1.969 & -.0000 & .7257 \\ 3.000 & 4.926 & 3.207 \end{bmatrix}$$

$$\underline{\underline{G}}_{10} = \begin{bmatrix} 2.579 & .0000 & -4.491 \\ 4.936 & .0000 & 2.061 \\ 7.556 & 1.232 & 9.103 \end{bmatrix}$$

$$\underline{\underline{G}}_{11} = \begin{bmatrix} 4.956 & -.0000 & -8.981 \\ 9.354 & .0000 & 4.121 \\ 14.24 & 23.40 & 18.20 \end{bmatrix}$$

$$\underline{\underline{G}}_{12} = \begin{bmatrix} 5.216 & .0000 & -9.474 \\ 9.846 & .0000 & 4.347 \\ 14.99 & 24.63 & 19.20 \end{bmatrix}$$

Table 4.9 (continued)

$$\underline{\underline{C}}_{13} = \begin{bmatrix} 2.608 & .1252 & -3.554 \\ 4.923 & .2378 & 1.632 \\ 7.497 & 15.14 & 7.219 \end{bmatrix} \qquad \underline{\underline{C}}_{14} = \begin{bmatrix} 4.661 & -1.557 & -2.552 \\ 8.885 & -2.969 & 1.156 \\ 13.58 & 10.23 & 5.151 \end{bmatrix}$$

$$\underline{\underline{C}}_{15} = \begin{bmatrix} 3.107 & 2.076 & -2.540 \\ 5.923 & 3.957 & 1.179 \\ 9.055 & 30.67 & 5.219 \end{bmatrix} \qquad \underline{\underline{C}}_{16} = \begin{bmatrix} 3.107 & 2.076 & -9.383 \\ 5.923 & 3.957 & 4.310 \\ 9.054 & 30.67 & 18.97 \end{bmatrix}$$

$$\underline{\underline{C}}_{17} = \begin{bmatrix} 5.176 & .0016 & -9.390 \\ 9.867 & .0030 & 4.297 \\ 15.08 & 24.62 & 18.95 \end{bmatrix} \qquad \underline{\underline{C}}_{18} = \begin{bmatrix} 5.178 & -2.076 & -2.554 \\ 9.871 & -3.957 & 1.152 \\ 15.09 & 8.721 & 5.146 \end{bmatrix}$$

$$\underline{\underline{C}}_{19} = \begin{bmatrix} 3.107 & -1.156 & -9.401 \\ 5.923 & -2.969 & 4.290 \\ 9.055 & 2.847 & 18.89 \end{bmatrix}$$

TABLE 4.10

Closed-loop Eigenvalues and Right Eigenvectors for Rosenbrock's Approximate Modal Control Method [8]

Run 1 - Eigenvalues:

0.502	0.607	0.657±0.269i	0.897

Corresponding Right Eigenvectors (columnwise):

-0.299	-0.082	-0.046±0.432i	0.592
-0.443	-0.084	-0.019±0.032i	-0.005
0.323	0.617	0.042±0.270i	-0.051
0.053	0.138	-0.035±0.306i	0.795
0.779	0.766	1.0 ±0.0 i	0.128

Run 2 - Eigenvalues:

0.287	0.625	0.697±0.201i	0.895

Corresponding Right Eigenvectors (columnwise):

0.745	0.036	-0.033±0.016i	0.005
0.295	0.074	0.144±0.440i	-0.609
0.106	-0.694	0.004±0.372i	0.055
-0.047	-0.147	0.066±0.358i	-0.780
0.587	-0.700	1.0 ±0.0 i	-0.136

Run 10 - Eigenvalues:

0.486	0.500	0.672±0.243i	0.894

Corresponding Right Eigenvectors (columnwise):

-0.287	-0.640	0.107±0.152i	-0.019
-0.914	-0.622	0.177±0.249i	-0.033
-0.288	-0.451	0.127±0.429i	-0.610
0.000	-0.000	-0.003±0.323i	-0.779
0.000	-0.000	1.0 ±0.0 i	-0.139

Table 4.10 (continued)

Run 11 - Eigenvalues:

0.050	0.050	0.625±0.431i	0.897

Corresponding Right Eigenvectors (columnwise):

0.015	0.520	0.117±0.088i	0.015
-0.993	0.763	0.192±0.143i	0.027
-0.120	0.385	-0.155±0.400i	0.582
0.000	0.000	-0.161±0.255i	0.803
0.000	0.000	1.0 ±0.0 i	0.127

Run 12 - Eigenvalues:

0.000	0.000	0.620±0.447i	0.898

Corresponding Right Eigenvectors (columnwise):

-0.038	0.878	0.117±0.085i	0.015
-0.988	-0.046	0.192±0.138i	0.026
-0.148	0.477	-0.169±0.392i	0.580
-0.000	0.000	-0.168±0.249i	0.804
0.000	0.000	1.0 ±0.0 i	0.126

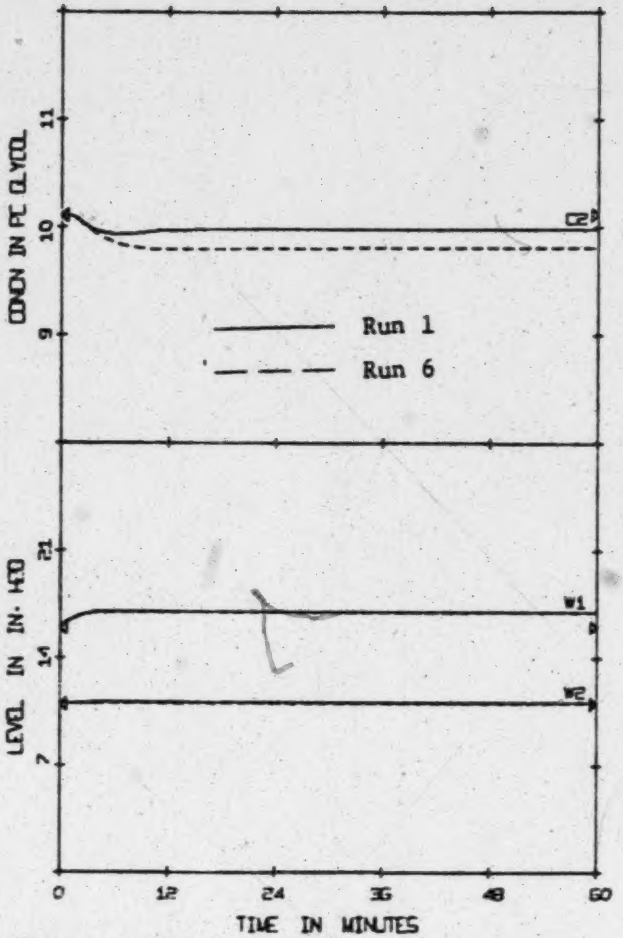

Figure 4.3 Effect of Eigenvector Ordering
(F Disturbance)

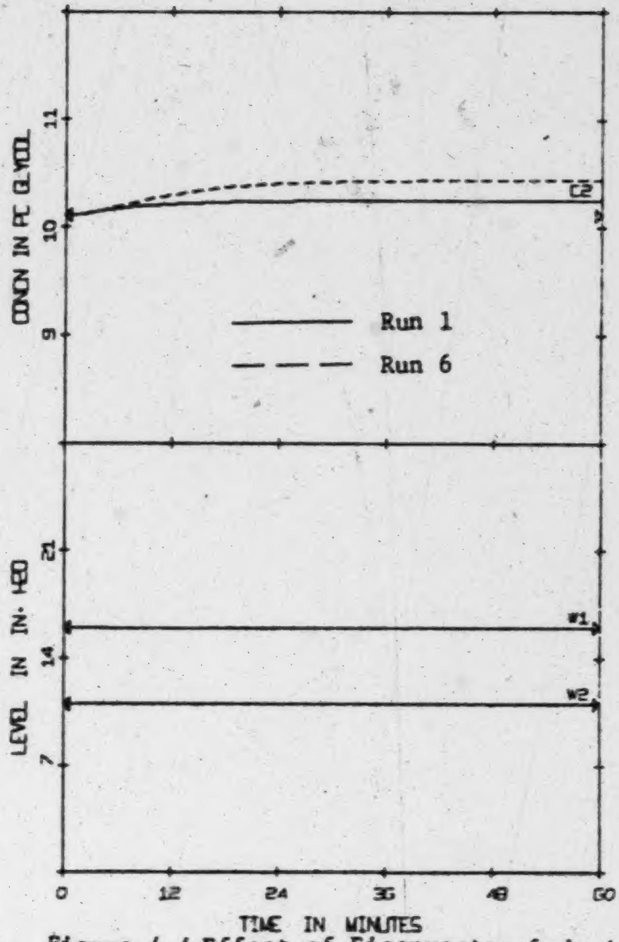

Figure 4.4 Effect of Eigenvector Ordering
(CF Disturbance)

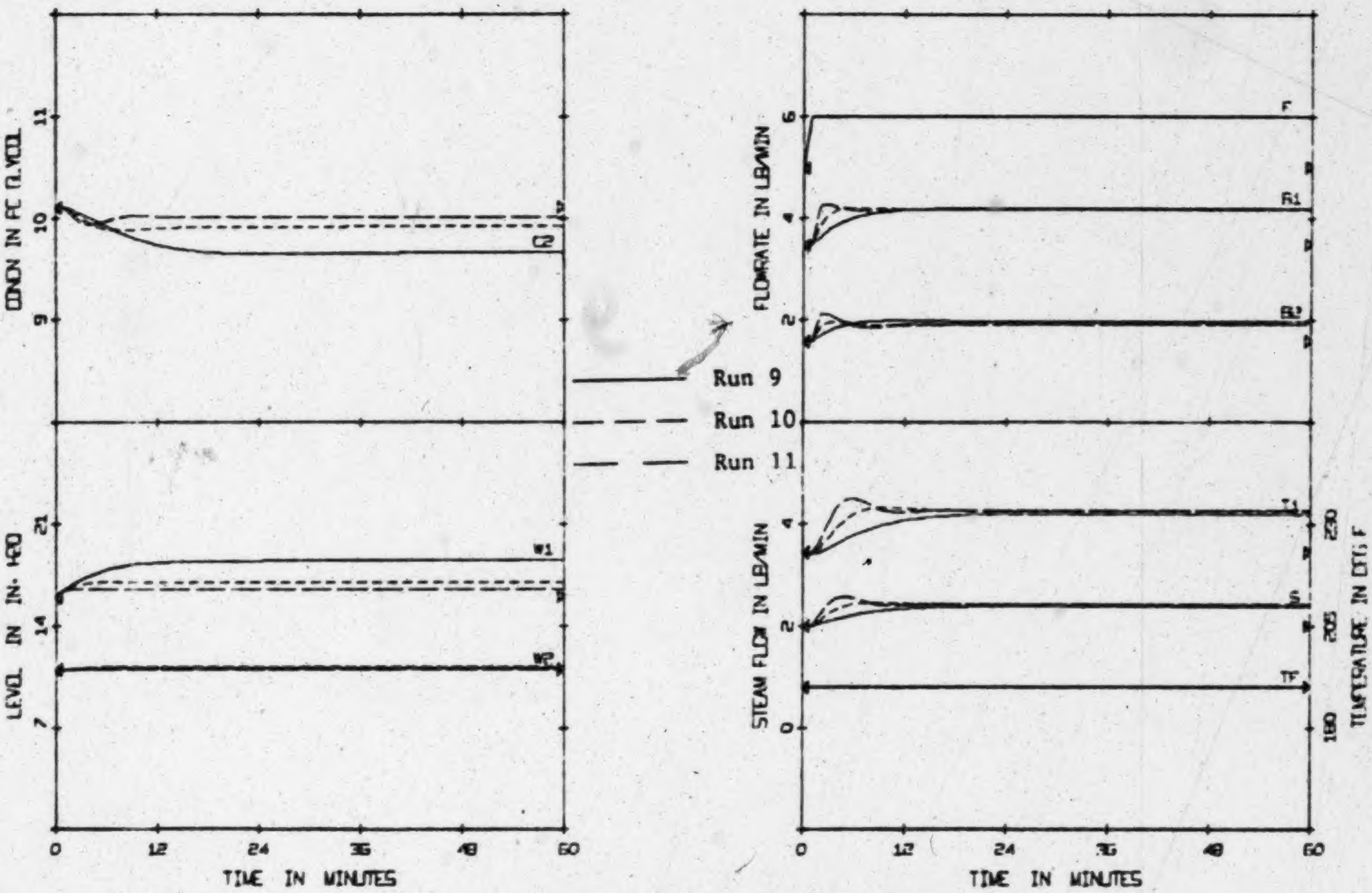

Figure 4.5 Effect of the Absolute Magnitude of Desired Eigenvalues (F Disturbance)

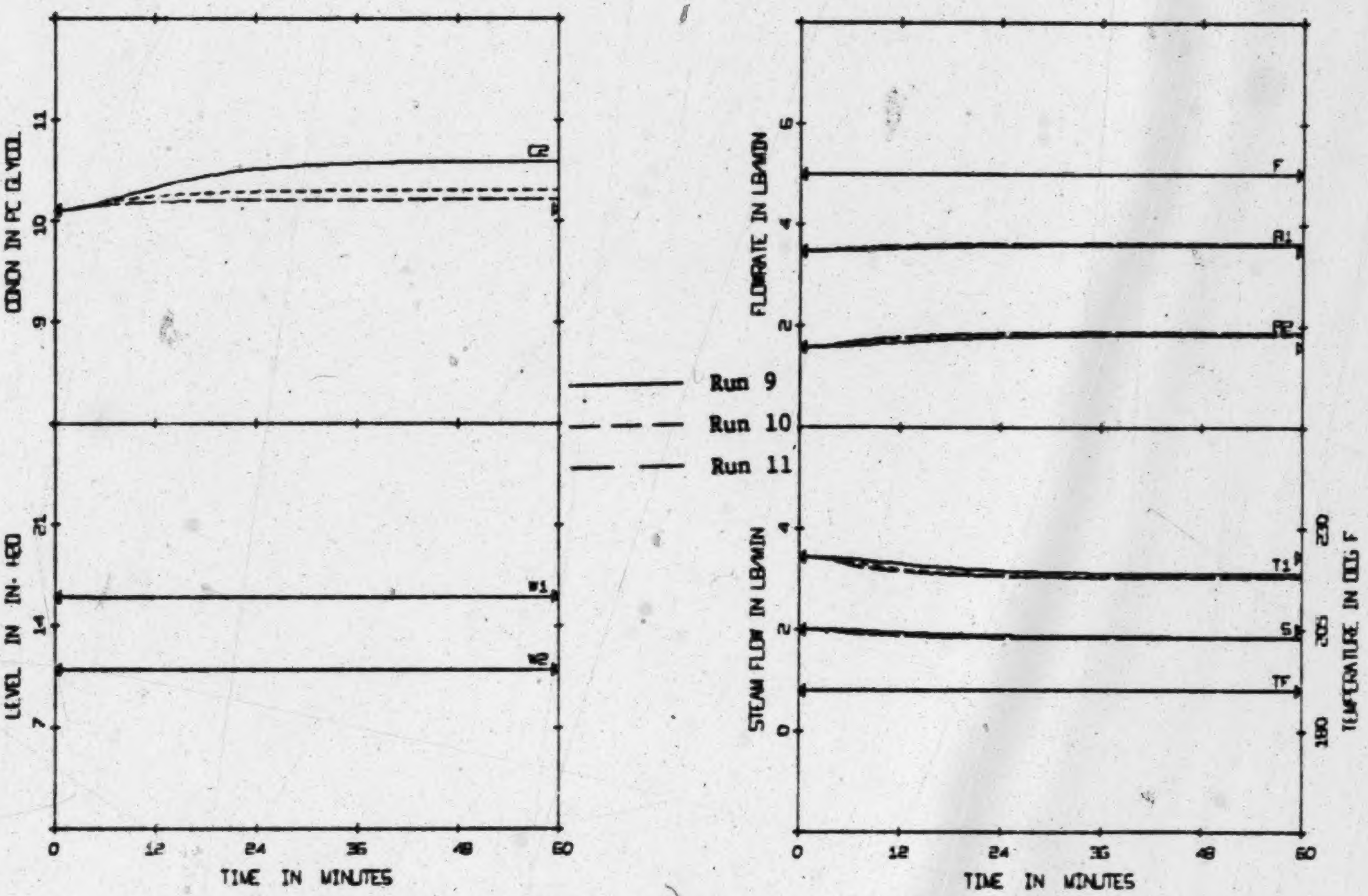

Figure 4.6 Effect of the Absolute Magnitude of Desired Eigenvalues (CF Disturbance)

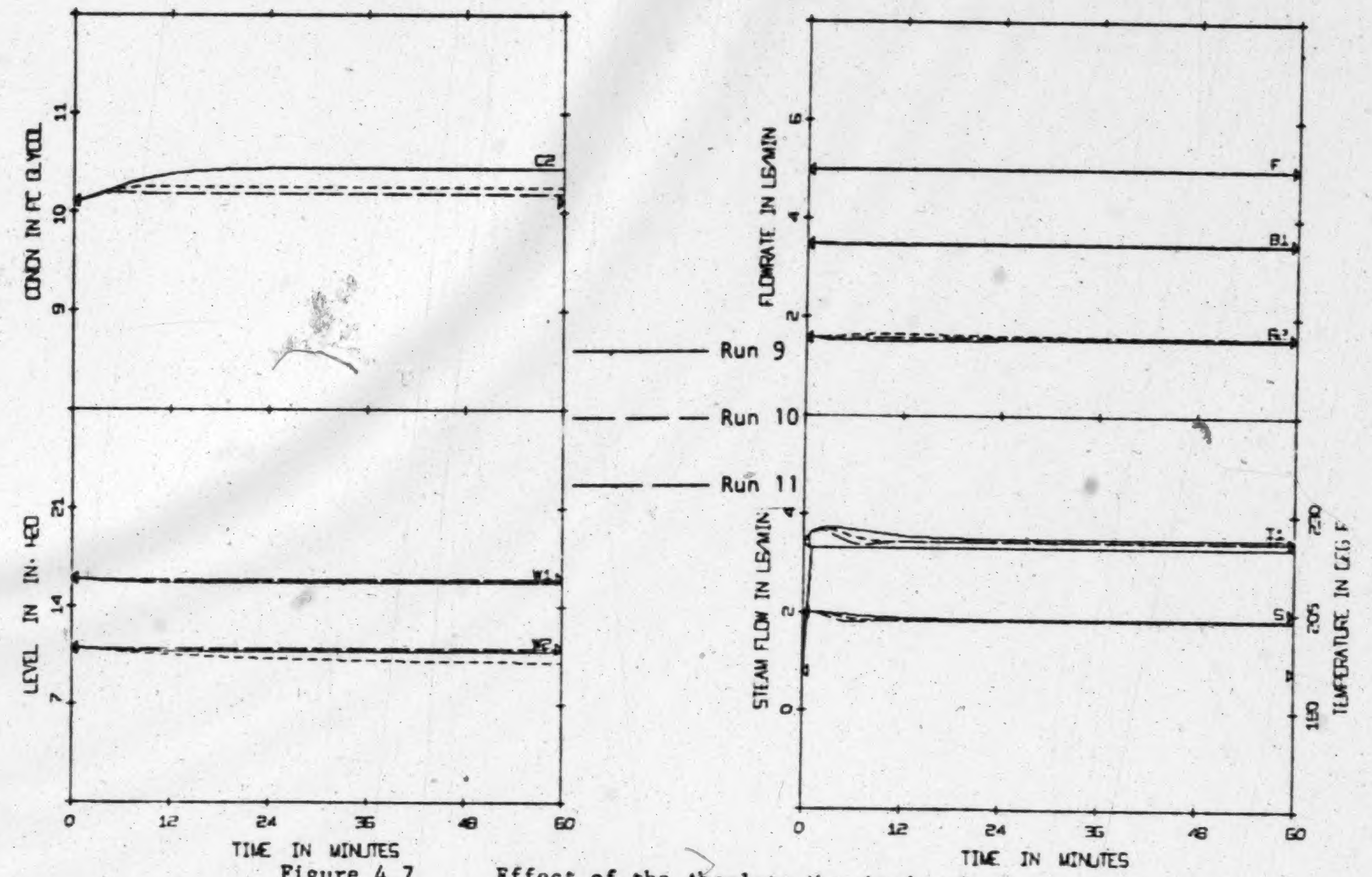

Figure 4.7 Effect of the Absolute Magnitude of Desired Eigenvalues (TF Disturbance)

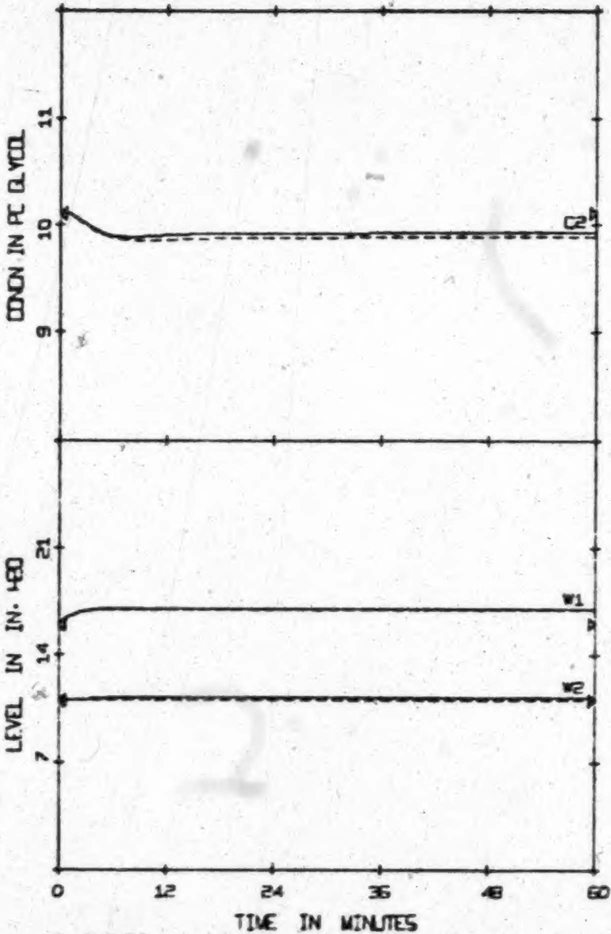

Figure 4.8 Effect of the Relative Magnitude of Desired Eigenvalues (F Disturbance)

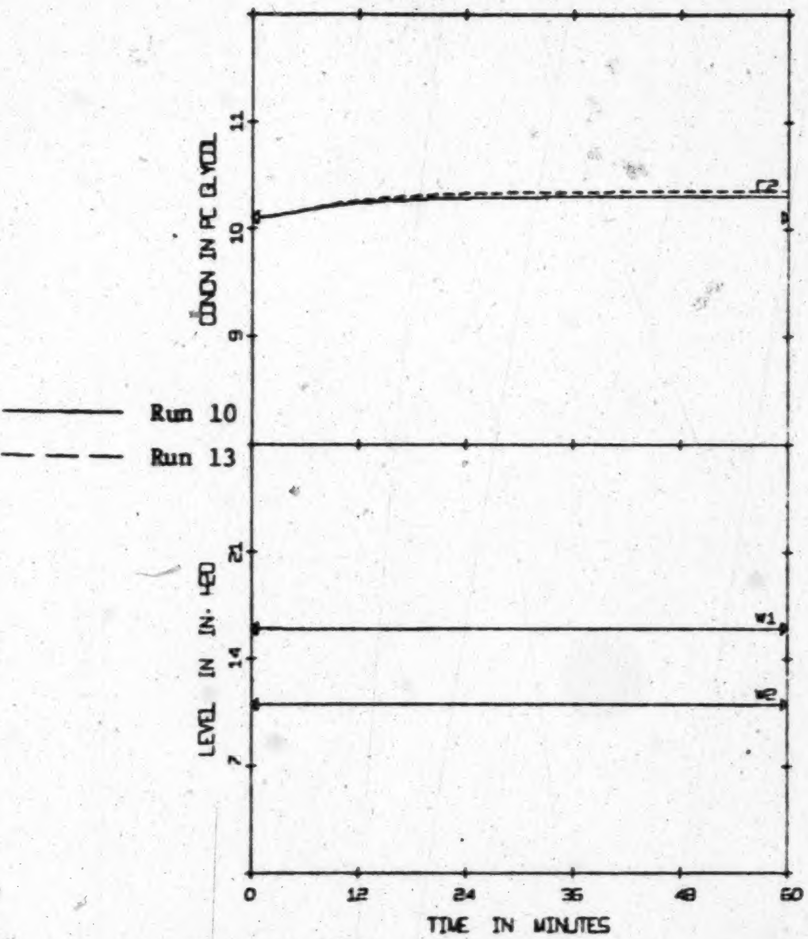

Figure 4.9 Effect of the Relative Magnitude of Desired Eigenvalues (CF Disturbance)

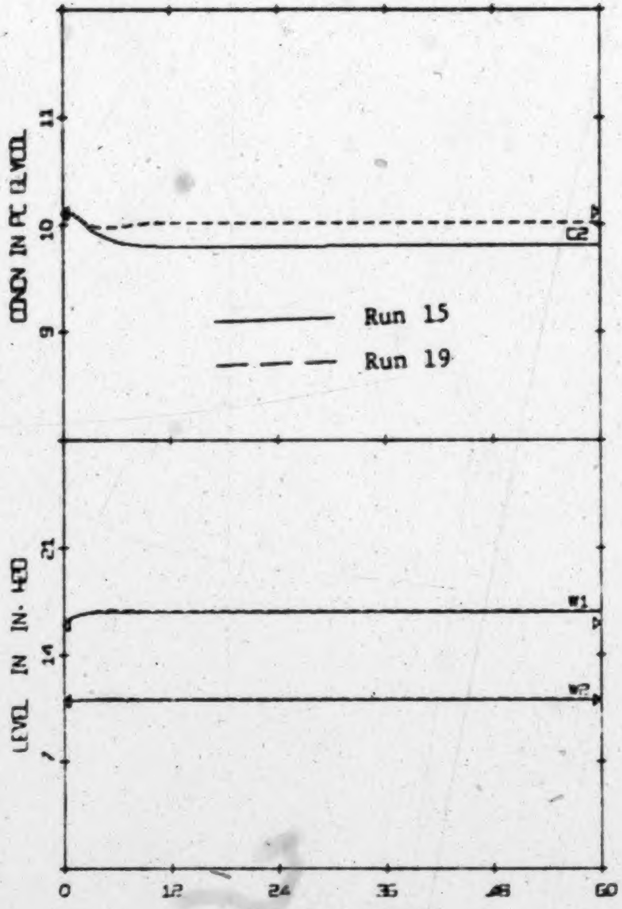

Figure 4.10 Effect of Eigenvalue Pairings (F Disturbance)

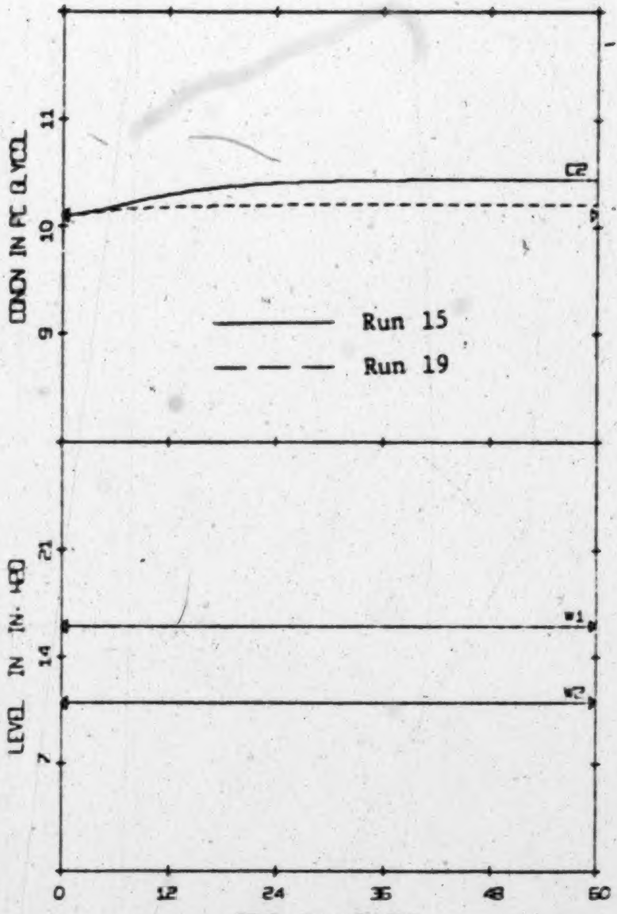

Figure 4.11 Effect of Eigenvalue Pairings (CF Disturbance)

TABLE 4.11

Design Specifications for Takahashi's Approximate Modal Control Method [9]

Run Number	Desired Eigenvalues				
20	0.07	0.06	0.05	0.04	0.03
21	0.00005	0.00004	0.00003	0.00002	0.00001
22	0.07	0.06	0.05	0.92155	0.43845
23	0.00005	0.00004	0.00003	0.92155	0.43845
24	0.07	0.06	0.05	0.04	0.03

TABLE 4.12

Response Characteristics for Takahashi's Approximate Modal Control Method [9]

Run Number	Corresponding Figures	Offset in C2' for F Disturbance	Offset in C2' for CF Disturbance
20	4.12, 4.13	-0.167	0.087
21		-0.162	0.085
22	4.12, 4.13	-0.091	0.049
23		-0.091	0.049
24	4.12, 4.13	-0.018	0.009

TABLE 4.13

Closed-loop Eigenvalues for Takahashi's Approximate Modal Control Method [9]

Run Number	Closed-loop Eigenvalues			
20	0.061	0.148	0.926±0.030i	0.466
21	0.000	0.088	0.924±0.030i	0.467
22	0.020	0.092	0.895±0.041i	0.507
23	0.014	0.086	0.895±0.041i	0.508
24			-	

TABLE 4.14

Controller Matrices for Takahashi's Approximate Modal Control Method [9]

$$\underline{G}_{20} = \begin{bmatrix} .7758 & -.2997 & -.4112 \\ 8.768 & -.1196 & 3.270 \\ 16.14 & 22.60 & 8.997 \end{bmatrix} \qquad \underline{G}_{21} = \begin{bmatrix} .8348 & -.3242 & -.4357 \\ 9.427 & -.1544 & 3.452 \\ 17.36 & 23.99 & 9.501 \end{bmatrix}$$

$$\underline{G}_{22} = \begin{bmatrix} 1.013 & -.0570 & -1.243 \\ 9.376 & -.1309 & 3.372 \\ 17.11 & 23.67 & 9.996 \end{bmatrix} \qquad \underline{G}_{23} = \begin{bmatrix} 1.020 & -.0576 & -1.249 \\ 9.441 & -.1342 & 3.390 \\ 17.23 & 23.81 & 10.05 \end{bmatrix}$$

$$\underline{G}_{24} = \begin{bmatrix} .7758 & -.2997 & -10.00 \\ 8.768 & -.1196 & 3.270 \\ 16.14 & 22.60 & 8.997 \end{bmatrix}$$

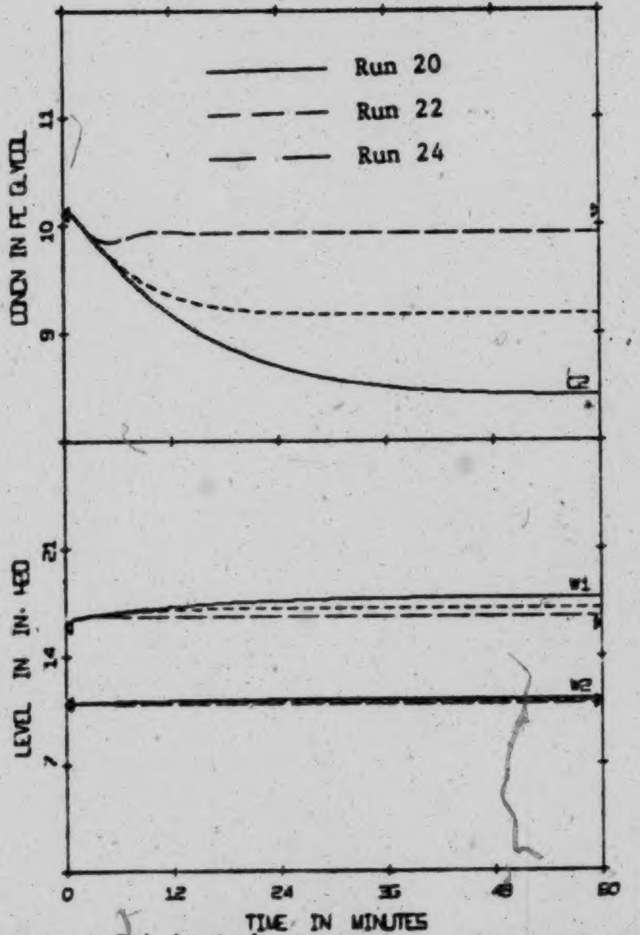

Figure 4.12 Takahashi's Approximate Modal Control Method (F Disturbance)

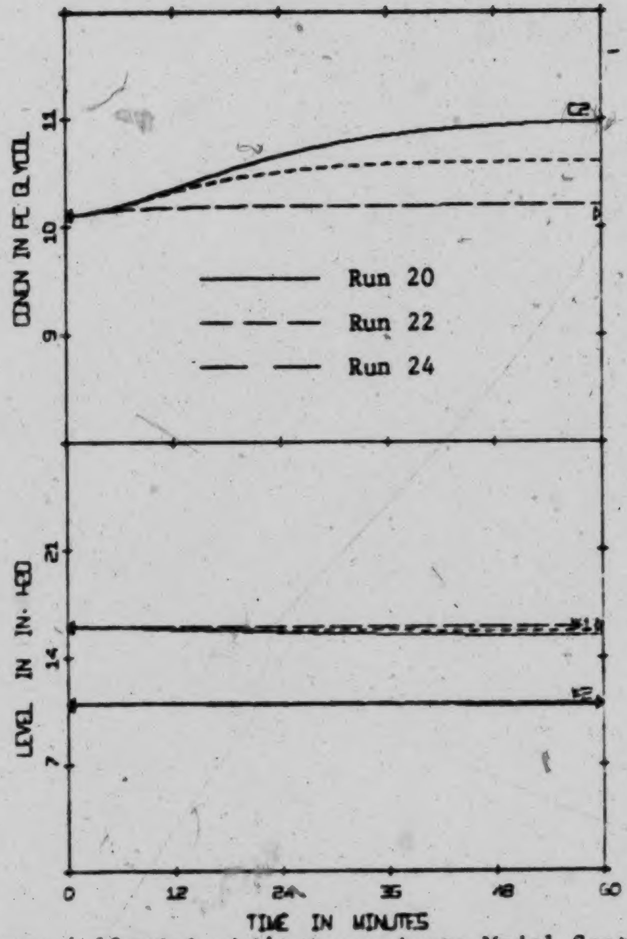

Figure 4.13 Takahashi's Approximate Modal Control Method (CF Disturbance)

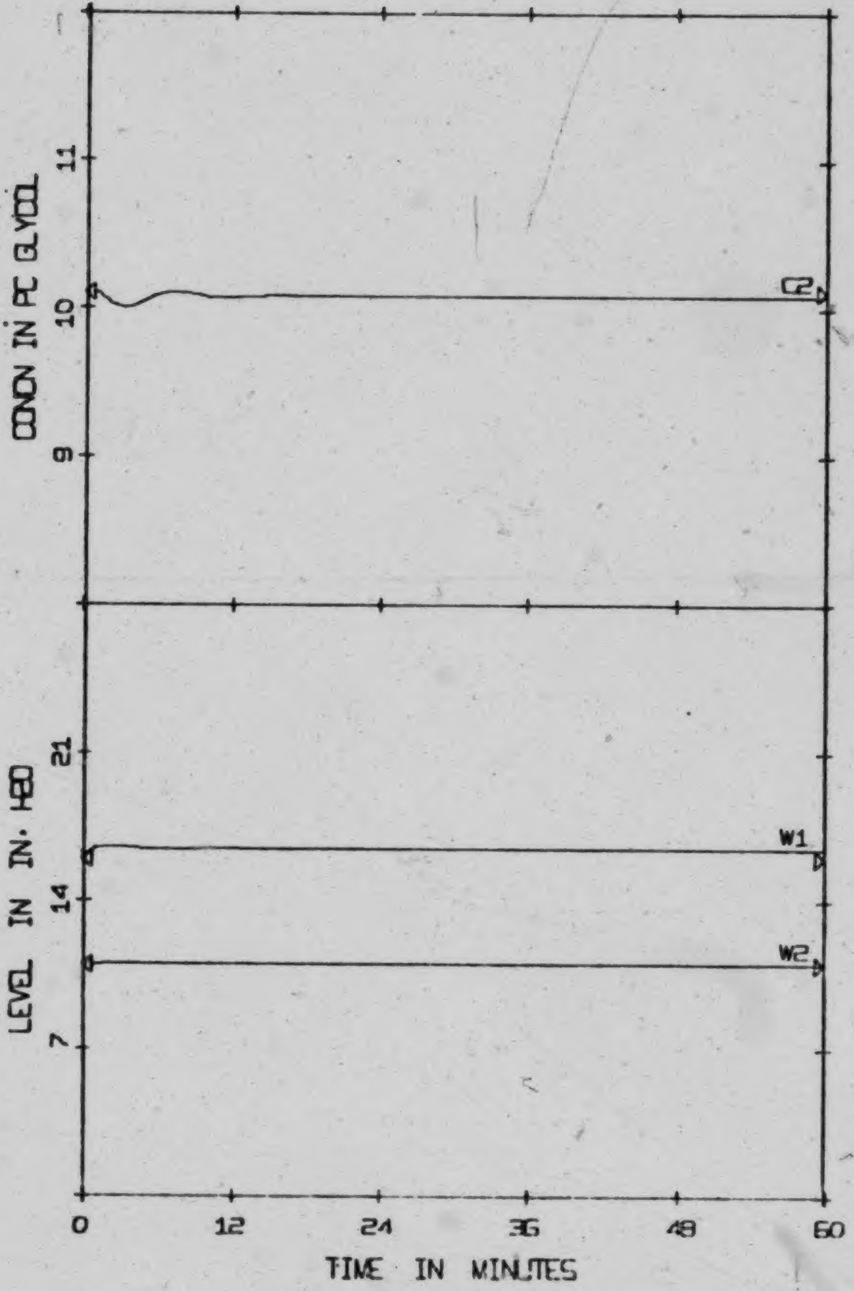

Figure 4.14 Model Reduction and Ideal Modal Control (F Disturbance)

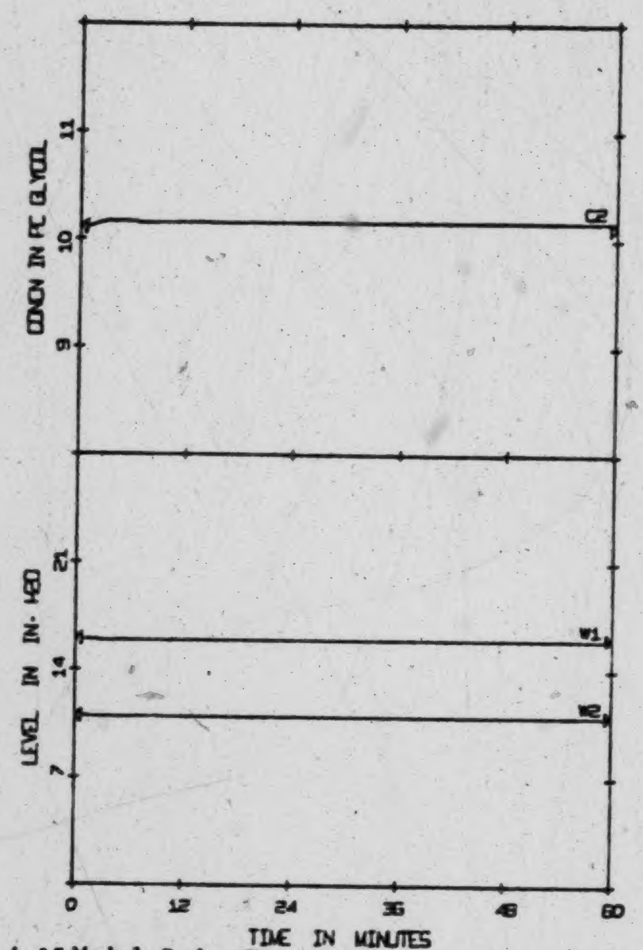

Figure 4.15 Model Reduction and Ideal Modal Control
(CF Disturbance)

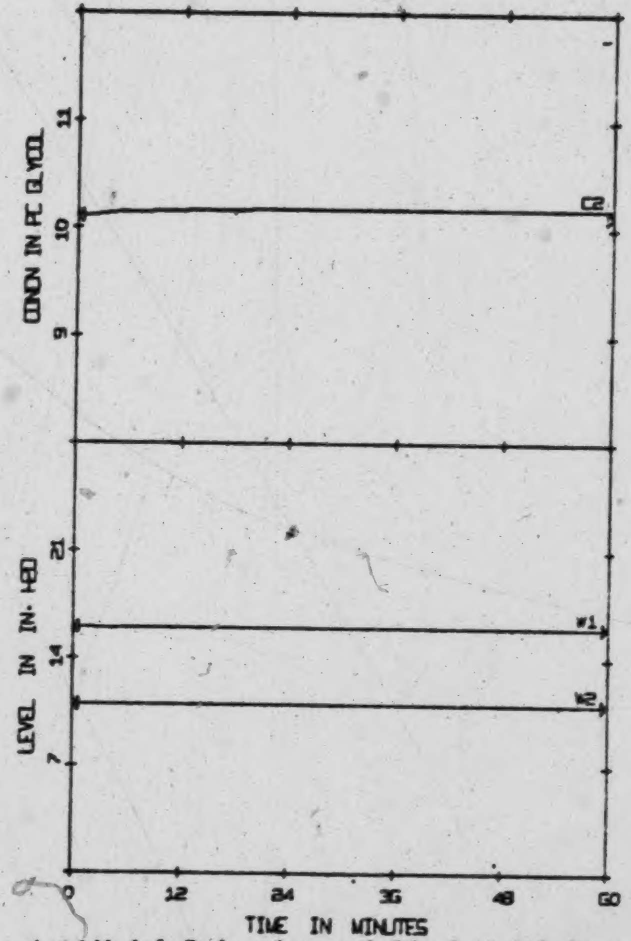

Figure 4.16 Model Reduction and Ideal Modal Control
(TF Disturbance)

TABLE 4.15

Design Specifications for Eigenvalue Assignment Methods

Run No.	No. of ℓ's		ℓ_1			ℓ_2			Desired Eigenvalues	
25	1	1.	1.	1.		-		0.3	0.4	0.5
26	1	.02	95.	.05		-		0.3	0.4	0.5
27	1	1.	1.	1.		-		0.0006	0.0007	0.5
28	2	1.	-1.	1.		-		0.0006	0.960	1.0
		1.	1.	1.		-		0.0006	0.0007	0.5
29	2	1.	1.	1.		-		0.6	0.7*	0.8*
					1.000	-1.873	-2.324	0.62	0.72	0.82
30	2	1.	1.	1.				0.6	0.7*	0.8*
					1.000	-1.873	-2.324	0.75	0.85	0.9
31	3	1.	-1.	1.				0.7	0.96	1.0
		1.	1.	1.				0.5	0.7*	0.8*
					-4.408	-1.007	1.000	0.62	0.72	0.82

Table 4.15 (continued)

Run No.	No. of g's	g_1			g_2			Desired Eigenvalues		
32	3	1.	-1.	1.				0.006	0.922	0.438
		1.	1.	1.				0.006*	0.007	0.438*
					2.444	-1.065	1.000	0.7	0.8	0.9
33	3	1.	-1.	1.				0.006	0.922	0.438
		1.	1.	1.				0.006*	0.007	0.438*
					2.444	-1.065	1.000	0.6	0.8	0.9
34	3	1.	-1.	1.				0.006	0.922	0.438
		1.	1.	1.				0.006*	0.007	0.438*
					2.444	-1.605	1.000	0.4	0.8	0.9
35	3	1.	-1.	1.				0.006	0.922	0.438
		1.	1.	1.				0.006*	0.007	0.438*
					2.444	-1.065	1.000	0.007	0.8	0.9

Table 4.15 (continued)

Run No.	No. of $\underline{g}$'s	$\underline{g}_1$			$\underline{g}_2$			Desired Eigenvalues		
36	4	1.	-1.	1.				0.1	0.922	0.438
		1.	1.	1.				0.1	0.2	0.438
		-1.	1.	-1.				0.1*	0.2*	0.3
					10.18	-0.920	1.000	0.7	0.8	0.9
37	4	1.	-1.	1.				0.1	0.922	0.438
		1.	1.	1.				0.1	0.2	0.438
		-1.	1.	-1.				0.1*	0.2*	0.3
					19.18	-0.920	1.000			
38	1	1.	1.	1.				0.6	0.7	0.8
39	3	1.	-1.	1.				0.1	0.922	0.438
		1.	1.	1.				0.1	0.2	0.438
		-1.	1.	-1.				0.1	0.2	0.438

Note: * designates the protected eigenvalues

TABLE 4.16

Response Characteristics for Eigenvalue Assignment Methods

Run Numbers	Corresponding Figures	Offset in C2' for F Disturbance	Offset in C2' for CF Disturbance
29	4.17, 4.18	-0.004	0.184
30	4.17, 4.18	0.010	0.201
31	4.17, 4.18	0.054	0.054
32	4.19, 4.20	0.090	0.085
33	4.19, 4.20	0.050	0.067
34	4.19, 4.20	-0.013	0.038
35	4.19, 4.20	-0.056	0.018
36	4.21, 4.22	-0.016	-0.003
37	4.23, 4.24	0.003	0.001
38	4.25, 4.26	unstable	unstable
39	4.25, 4.26	-0.004	0.086

TABLE 4.17

Actual Closed-loop Eigenvalues for the Eigenvalue Assignment Methods

Run Number	Closed-loop Eigenvalues				
25	0.3	0.4	0.5	1.0	1.0
26	0.3	0.4	0.5	1.0	1.0
27	0.0006	0.0007	0.5	1.0	1.0
28	0.006	0.007	0.5	0.929±0.073i	
29	0.7	0.8	0.62	0.72	0.82
30	0.7	0.8	0.75	0.85	0.9
31	0.7	0.8	0.62	0.72	0.82
32	0.006	0.438	0.7	0.8	0.9
33	0.006	0.438	0.6	0.8	0.9
34	0.006	0.438	0.4	0.8	0.9
35	0.006	0.438	0.007	0.8	0.9
36	0.1	0.2	0.7	0.8	0.9
37	0.1	0.2	0.7	0.850±0.075i	
38	0.6	0.7	0.8	0.97	1.0
39	0.1	0.2	0.3	0.951±0.051i	

TABLE 4.18

Controller Matrices

$$\underline{\underline{G}}_{25} = \begin{bmatrix} 8.204 & -170.7 & -163.4 \\ 8.204 & -170.7 & -163.4 \\ 8.204 & -170.7 & -163.4 \end{bmatrix} \qquad \underline{\underline{G}}_{26} = \begin{bmatrix} .0116 & -.0123 & -.0401 \\ 55.13 & -58.26 & -190.4 \\ .0290 & -.0307 & -1.002 \end{bmatrix}$$

$$\underline{\underline{G}}_{27} = \begin{bmatrix} 12.13 & -453.9 & -444.4 \\ 12.13 & -453.9 & -444.4 \\ 12.13 & -453.9 & -444.4 \end{bmatrix} \qquad \underline{\underline{G}}_{28} = \begin{bmatrix} 2.344 & 16.79 & 22.05 \\ 19.86 & 14.77 & 22.05 \\ 2.344 & 16.79 & 22.05 \end{bmatrix}$$

$$\underline{\underline{G}}_{29} = \begin{bmatrix} .3066 & 22.11 & 16.60 \\ 7.637 & -66.30 & -55.51 \\ 8.884 & -80.18 & -66.83 \end{bmatrix} \qquad \underline{\underline{G}}_{30} = \begin{bmatrix} 2.187 & -2.268 & -4.354 \\ 2.965 & -20.64 & -16.27 \\ 3.087 & -23.52 & -18.14 \end{bmatrix}$$

$$\underline{\underline{G}}_{31} = \begin{bmatrix} 13.42 & 13.43 & 20.80 \\ 6.550 & 5.234 & 10.76 \\ -.2954 & 4.914 & 4.853 \end{bmatrix} \qquad \underline{\underline{G}}_{32} = \begin{bmatrix} 25.00 & 33.23 & 52.60 \\ 4.009 & .9010 & 5.319 \\ 12.07 & 23.66 & 33.15 \end{bmatrix}$$

$$\underline{\underline{G}}_{33} = \begin{bmatrix} 19.11 & 26.45 & 39.87 \\ 6.575 & 3.955 & 10.87 \\ 9.659 & 20.88 & 27.94 \end{bmatrix} \qquad \underline{\underline{G}}_{34} = \begin{bmatrix} 4.464 & 9.597 & 8.204 \\ 12.96 & 11.20 & 24.67 \\ 3.666 & 13.99 & 14.98 \end{bmatrix}$$

$$\underline{\underline{G}}_{35} = \begin{bmatrix} -16.69 & -14.75 & -37.53 \\ 22.18 & 21.81 & 44.61 \\ -4.991 & 4.022 & -3.733 \end{bmatrix} \qquad \underline{\underline{G}}_{36} = \begin{bmatrix} 1.664 & -6.485 & -8.227 \\ 7.538 & 10.49 & 25.93 \\ 5.786 & 17.60 & 29.36 \end{bmatrix}$$

Table 4.18 (continued)

$$\underline{G}_{37} = \begin{bmatrix} 10.65 & 5.436 & 12.20 \\ 6.725 & 9.359 & 24.08 \\ 6.667 & 18.84 & 22.37 \end{bmatrix} \qquad \underline{G}_{38} = \begin{bmatrix} 2.458 & -8.662 & -8.500 \\ 2.458 & -8.662 & -8.500 \\ 2.458 & -8.662 & -8.500 \end{bmatrix}$$

$$\underline{G}_{39} = \begin{bmatrix} 6.235 & 20.30 & 21.27 \\ 7.125 & 8.015 & 23.26 \\ 6.235 & 20.30 & 23.26 \end{bmatrix}$$

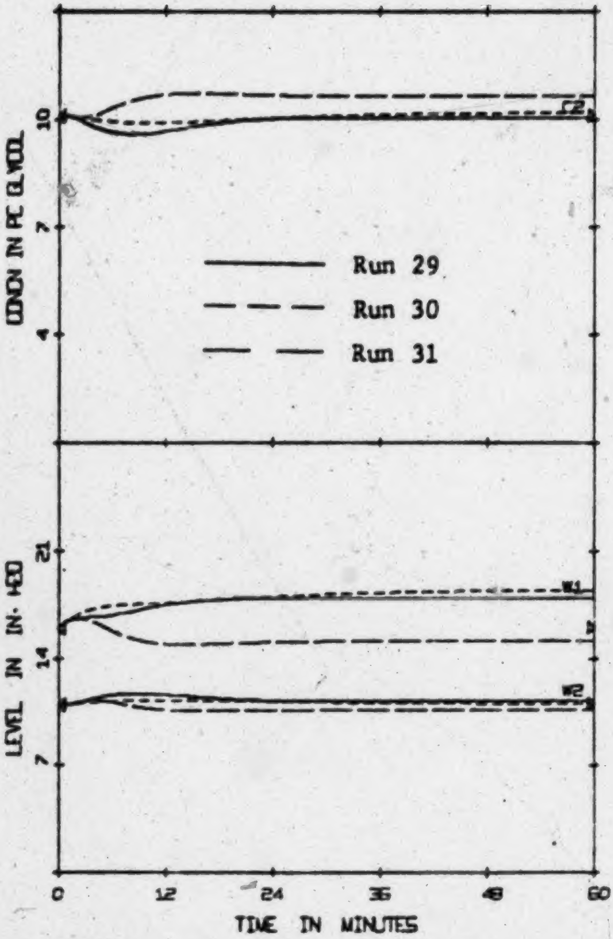

Figure 4.17 Effect of Desired Eigenvalues and Number of Recursive Steps (F Disturbance)

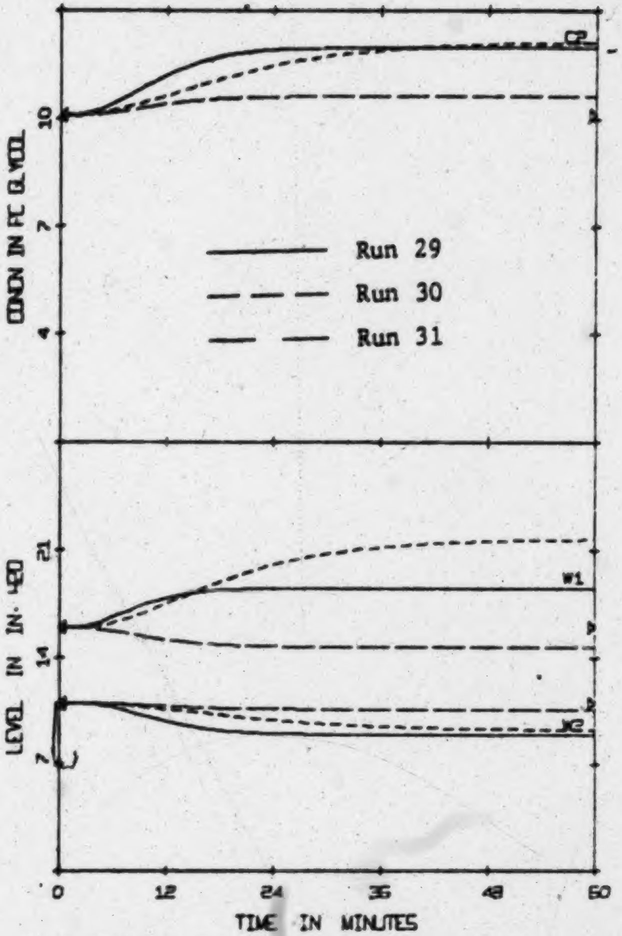

Figure 4.18 Effect of Desired Eigenvalues and Number of Recursive Steps (CF Disturbance)

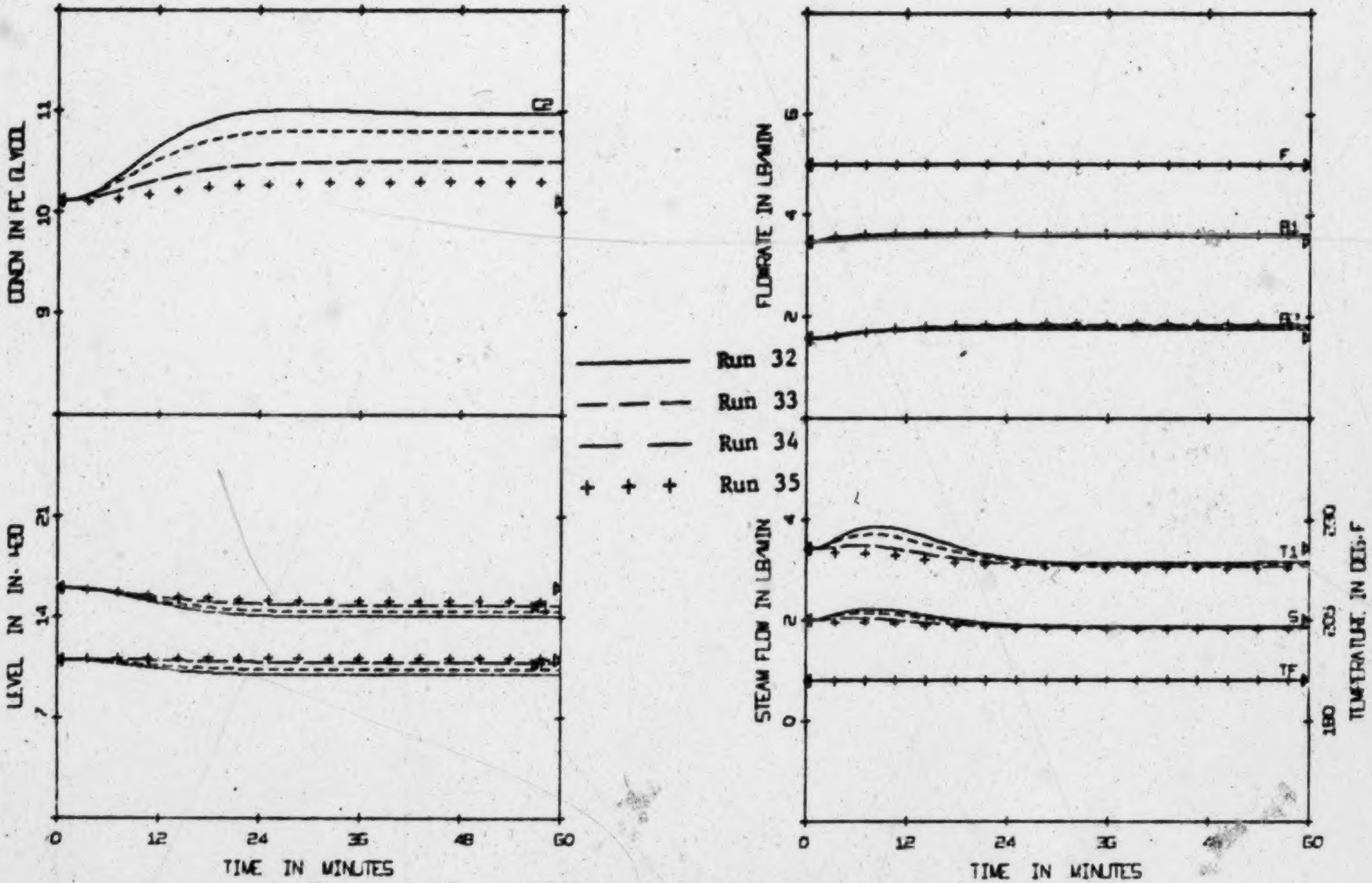

Figure 4.19 Effect of the Variation of One Desired Eigenvalue (F Disturbance)

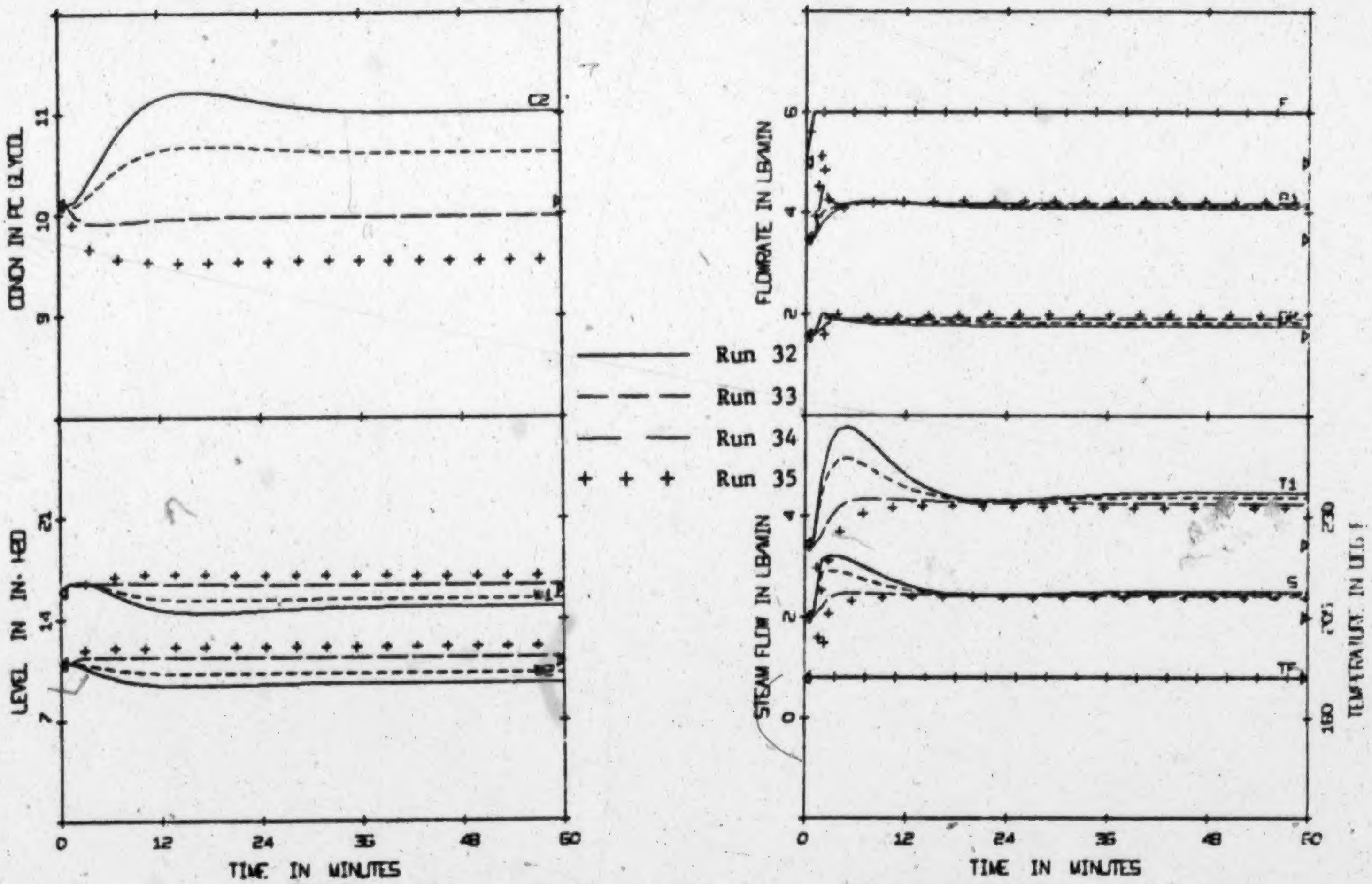

Figure 4.20 Effect of the Variation of One Desired Eigenvalue (CF Disturbance)

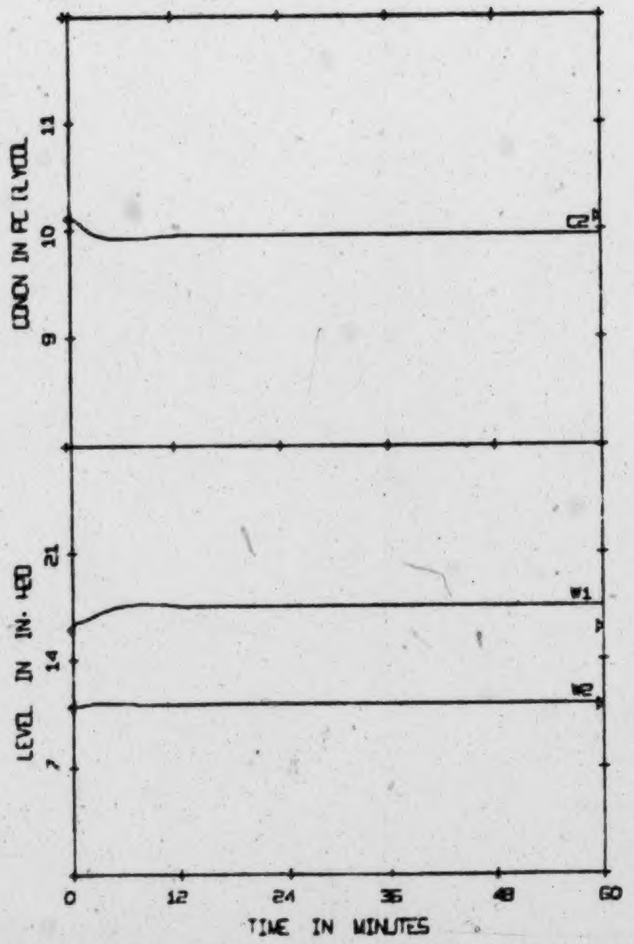

Figure 4.21 A Successful Design, Real Closed-loop Eigenvalues (F Disturbance)

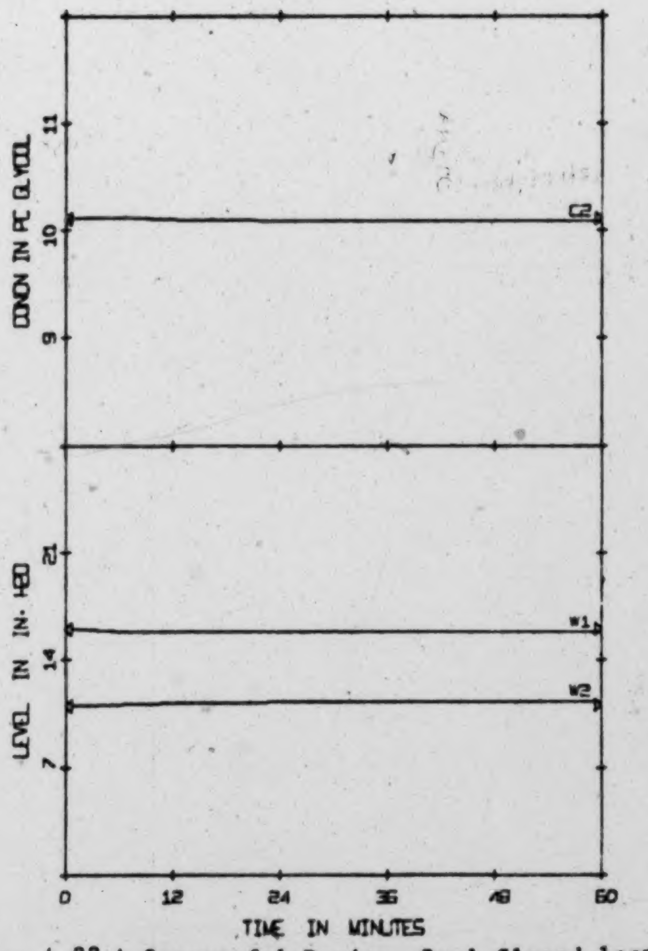

Figure 4.22 A Successful Design, Real Closed-loop Eigenvalues (CF Disturbance)

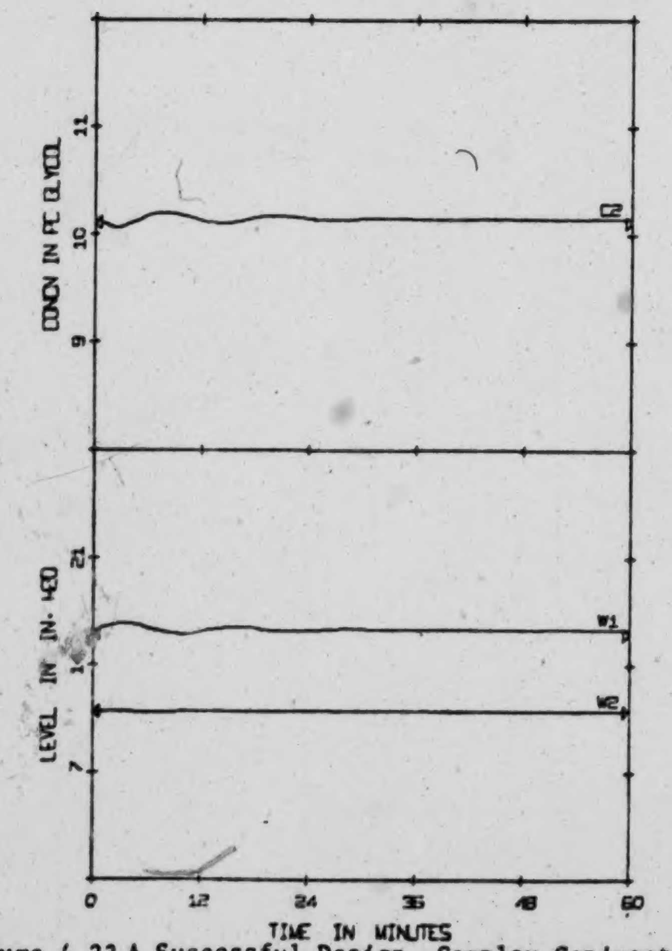

Figure 4.23 A Successful Design, Complex-Conjugate Closed-loop Eigenvalues (F Disturbance)

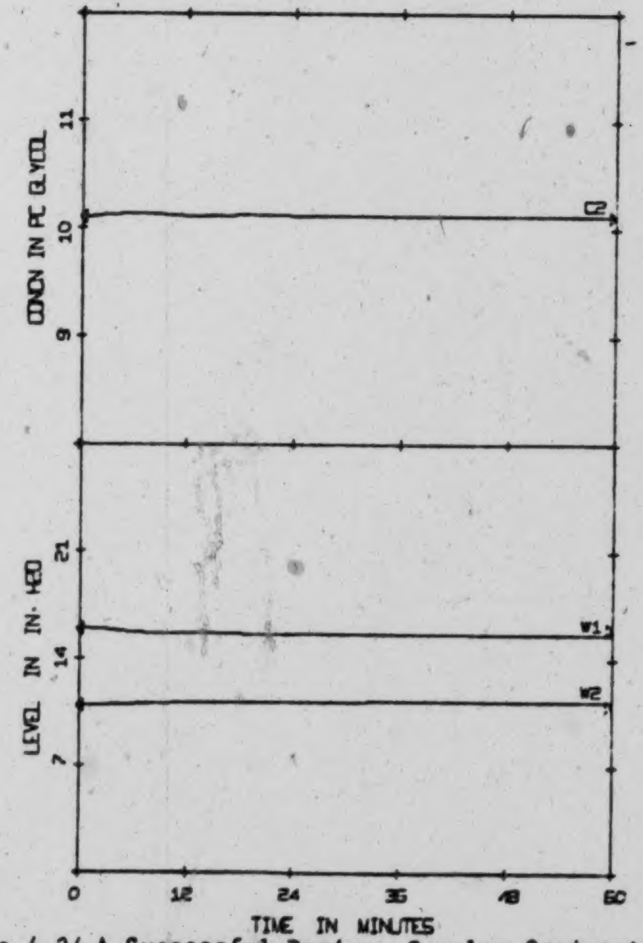

Figure 4.24 A Successful Design, Complex-Conjugate Closed-loop Eigenvalues (CF Disturbance)

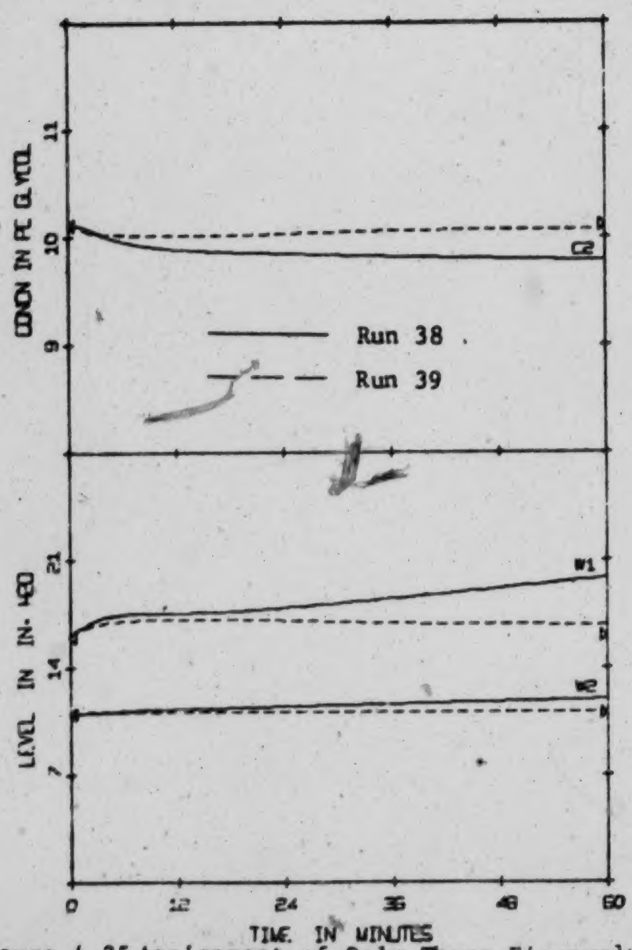

Figure 4.25 Assignment of Only Three Eigenvalues
(F Disturbance)

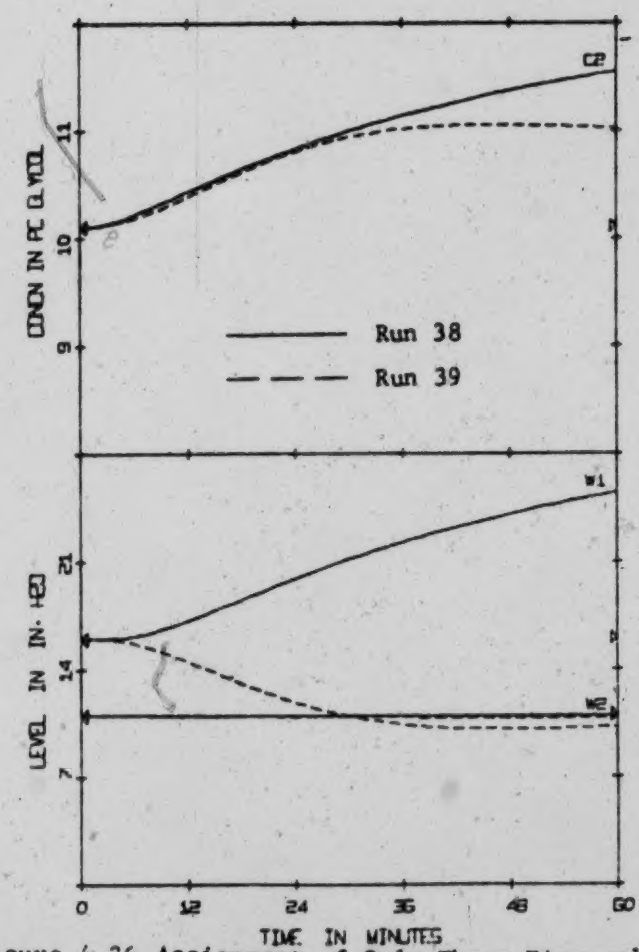

Figure 4.26 Assignment of Only Three Eigenvalues
(CF Disturbance)

CHAPTER FIVE

CONCLUSIONS

5.1 Interpretation of the Simulation Results

The different response characteristics observed in the application of eigenvalue assignment methods and Rosenbrock's approximate modal control method may be explained by the following arguments.

Consider the dyadic representation of the open-loop system matrix in terms of its eigenproperties, namely, $\underline{\underline{\Phi}} = \sum_{i=1}^{n} \lambda_i \, \underline{w}_i >< v_i$. The recursive application of the eigenvalue assignment method of Chapter Three to assign $\ell = \min(m,r)$ system eigenvalues will result in the following closed-loop system matrix, $\underline{\underline{\Phi}}_{CL}$,

$$\underline{\underline{\Phi}}_{CL} = \sum_{i=1}^{n} \lambda_i \, \underline{w}_i >< v_i + \sum_{i=1}^{\ell} \underline{g}_i >< \underline{k}_i \quad . \tag{5.1}$$

The directions of the $\{\underline{k}_i\}$ in (5.1) depend on the set of desired closed-loop eigenvalues, whereas the $\{\underline{g}_i\}$ can be assigned almost any set of elements. Thus, the closed-loop eigenvectors will, in general, be different from the open-loop ones and will highly depend on the choice of desired eigenvalues and the $\{\underline{g}_i\}$.

Application of Rosenbrock's approximate modal control method for the same purpose, on the other hand, will result in the following closed-loop system matrix (cf., (2.61)):

$$\underline{\underline{\Phi}}_{CL} = \sum_{i=1}^{n} \lambda_i \, \underline{w}_i >< v_i + (\underline{\underline{V}}_\ell^T \, \underline{\underline{B}})^{-1} \, \underline{\underline{K}} \, (\underline{\underline{C}} \, \underline{\underline{W}}_\ell)^{-1}$$

or

$$\underline{\underline{\Phi}}_{CL} = \sum_{i=1}^{n} \lambda_i \, \underline{w}_i \rangle\langle \underline{v}_i + \sum_{i=1}^{\ell} k_i \, \hat{\underline{h}}_i \rangle\langle \hat{\underline{f}}_i \qquad (5.2)$$

where $\hat{\underline{h}}_i$ represents the i^{th} column of $(\underline{\underline{V}}_\ell^T \, \underline{\underline{B}})^{-1}$ and $\hat{\underline{f}}_i$ represents the i^{th} row of $(\underline{\underline{C}} \, \underline{\underline{W}}_\ell)^{-1}$. $\hat{\underline{h}}_i$ and $\hat{\underline{f}}_i$ do not depend on the set of desired closed-loop eigenvalues, but k_i does. Therefore, for each set of desired closed-loop eigenvalues, in general, a different set of closed-loop eigenvectors will be obtained. But, as λ_{di} $(i = 1,2,\ldots,\ell)$ approaches zero, k_i $(i = 1,2,\ldots,\ell)$ will attain the value of the open-loop eigenvalues which are being shifted (cf., 2.32). Thus, the second summation in (5.2), namely the controller matrix, will become a constant matrix which in turn means that the closed-loop system eigenvectors will assume fixed directions in the state space. Thus, in Rosenbrock's approximate modal control method [8], the sensitivity of the eigenvectors to the specification of the desired closed-loop eigenvalues diminishes when λ_{di} $(i = 1,\ldots,\ell)$ approaches zero and, thus uniformly changing response characteristics which approach an asymptotic limit are observed (cf., Runs 9-12).

These observations cannot be made in the application of eigenvalue assignment techniques where the magnitude of the elements in $\{\underline{k}_i\}$ of (5.1) may become unbounded as the desired eigenvalues are made smaller. Even for the same specification of the $\{\underline{g}_i\}$, the eigenvector directions will be, in general, highly affected by the choice of the desired closed-loop eigenvalues, and thus significantly varying response characteristics can result, (cf., Runs 32-35). Similarly, since the eigenvector directions highly depend on the specification of the desired closed-loop eigenvalues, different response characteristics for CF and F disturbances are observed in different runs, i.e., for some runs, CF disturbances cause larger offsets and in other runs F disturbances are more severe.

This then forms an explanation of some of the phenomena which could not be accounted for in Chapter Four.

5.2 Comparison of Design Methods

A rather detailed discussion and comparison of the various modal control and eigenvalue assignment methods considered in this thesis have been provided in Chapter Two. Thus, this section will only summarize the experience gained with the four methods simulated in Chapter Four.

Both modal control and eigenvalue assignment methods increased the degree of stability and the speed of response of the system under consideration. The design options and design parameters involved in eigenvalue assignment methods were found to have a larger effect on the shape of the closed-loop system response than those involved in modal control methods. But currently there is no systematic way of exploiting these design options and parameters in order to improve the shape of response.

Application of Rosenbrock's approximate modal control method resulted in favorable response characteristics provided that the desired real eigenvalues had reasonably small magnitudes. The amount of offset in the controlled variables decreased as the real parts of the closed-loop eigenvalues decreased. The closed-loop eigenvectors tended to assume fixed directions in the state space as the desired real eigenvalues tended towards zero. Thus, the shape of response could not be affected to any great extent. The design options consisting of the ordering of the eigenvectors in the eigenvector matrices and the pairing of closed-loop and open-loop eigenvalues influenced the response characteristics of the system, but to a smaller extent than the location of the closed-loop eigenvalues. The gains in the controller matrices had the physically expected signs and magnitudes, were always reasonably small, and did not vary significantly for the various options considered. Different disturbances had a similar effect on the closed-loop response characteristics of the system for the same control law. The successful application of Rosenbrock's approximate modal control technique in this case study was attributed to some of the desirable features of the open-loop eigenproperties.

Application of the eigenvalue assignment algorithm of Chapter Three resulted in a wide spectrum of closed-loop response characteristics. The closed-loop eigenvector directions and the shape of responses were quite sensitive to the existing design options and design parameters. Thus, extensive utilization of these design options and parameters resulted in satisfactory controllers. The eigenvector directions and the response characteristics of the closed-loop system could not be correlated with the various design options and design parameters. The controller gains required for the assignment of a set of eigenvalues were always larger in absolute value than those resulting from the application of Rosenbrock's approximate modal control method. Recursive application of the eigenvalue assignment methods was found to reduce the controller gains and to improve the response characteristics. Some suggestions were made in Chapter Four for the successful application of this procedure. Different disturbances had a vastly different effect on the closed-loop response characteristics of the system.

Application of Takahashi's approximate modal control method gave large offsets in the controlled variables which could not be reduced greatly by the specification of smaller real desired closed-loop eigenvalues. The combination of a mode-based model reduction technique with the ideal modal control method, on the other hand, gave excellent results. The generality of the results noted in this paragraph is rather doubtful, though, since extensive studies were not performed to evaluate these last two approaches.

It is worth noting that approximate modal control techniques, in general, cannot guarantee the assignment of a set of eigenvalues. In fact,their application to some systems may even result in a less desirable set of closed-loop eigenvalues than the open-loop ones. Their success in this case study is due to the favorable eigenproperties of the open-loop system. By contrast the eigenvalue assignment method of Chapter Three guarantees the assignment of at least a subset of the system eigenvalues. Thus, its applicability is greater, and as demonstrated in Chapter Four it can provide excellent results if the available design options and parameters are properly exploited and the unassigned closed-loop eigenvalues do not attain undesirable locations in the complex plane.

The computational efforts involved in designing controllers is relatively small in both modal control and eigenvalue assignment methods.

5.3 Recommendations

The simulation studies in this thesis involve a relatively low order model. Therefore, it has not been possible to demonstrate the usefulness of modal analysis in configurating the control system. Similarly, it has not been possible to fully demonstrate the benefits of the new eigenvalue assignment method which in most instances guarantees the assignment of a significantly larger number of eigenvalues than in previously developed methods. Future work aiming to assess the real benefits of modal control and eigenvalue assignment should involve systems with higher order state space models.

The greater importance of the closed-loop eigenvector directions on the closed-loop response characteristics of the system became readily apparent during this investigation. It was realized that eigenvalue assignment did not provide satisfactory response characteristics unless extensive tuning was performed. Thus, systematic methods to affect the modes of the system rather than just its eigenvalues are greatly needed. The potential usefulness of the design freedom available in choosing the $\underline{g}$ vector in fulfilling this objective is undeniable. Preservation of the open-loop eigenvector directions in the closed-loop system is an arbitrary and unjustified objective unless the open-loop eigenvectors already have desirable directions.

Exact eigenvalue assignment has been found to be an unnecessarily ambitious goal. Rather than exactly assigning a few eigenvalues, future work should attempt to gain some control over all of the system eigenvalues.

Studies concerning determination of desirable closed-loop eigenvector directions for a specific system to fulfill such objectives as disturbance rejection, high integrity, and low sensitivity to controller gain variations may prove to be very fruitful.

NOMENCLATURE

Alphabetic

a	coefficient in a characteristic polynomial
$\underline{\underline{A}}$	system matrix of a continuous-time model
b	coefficient in a characteristic polynomial
$\underline{b}$	control coefficient vector
$\underline{\underline{B}}$	control coefficient matrix
B1	first effect bottoms flow
B2	second effect bottoms flow
$\underline{c}$	output coefficient vector
$\underline{\underline{C}}$	output coefficient matrix
C1	first effect concentration
C2	second effect concentration
CF	feed concentration
$\underline{d}$	disturbance vector
$\underline{\underline{D}}$	disturbance coefficient matrix
$\underline{e}$	vector defined in (3.18)
$\underline{\underline{E}}$	matrix defined in (3.25)
$\underline{f}$	column vector of $\underline{\underline{F}}$
F	feed concentration
$\underline{\underline{F}}$	modal output coefficient matrix
$\underline{\underline{F}}(\ )$	matrix function defined in (3.5)
$\underline{\underline{F}}_i$	dyadic matrix
$\underline{g}$	column vector of a dyadic controller
$\underline{\underline{G}}$	controller matrix

$\underline{h}$	column vector of $\underline{\underline{H}}$
$\underline{\underline{H}}$	modal control coefficient matrix
H1	first effect enthalpy
HF	feed enthalpy
J_1	objective function defined in (2.72)
J_2	objective function defined in (2.73)
k	controller gain
$\underline{k}$	controller vector
$\underline{\underline{K}}$	modal domain controller matrix
ℓ	number of eigenvalues assigned, number of modes controlled
L	weighting on eigenvalues
m	number of outputs
$\underline{m}$	mode vector
M	weighting on controller gains
n	number of states
p	number of disturbances
$\underline{p}$	column of matrix $\underline{\underline{P}}$
$\underline{\underline{P}}$	analyzer matrix
q( )	open-loop characteristic polynomial
$\underline{q}$	vector defined in (2.26)
$\underline{\underline{Q}}$	controllability matrix
r	number of controls
r( )	closed-loop characteristic polynomial
$\underline{r}$	column of $\underline{\underline{R}}$ matrix
$\underline{\underline{R}}$	synthesizer matrix

s	modal control element
$\underline{s}$	modal control vector
$\underline{\underline{S}}$	observability matrix
t	modal disturbance element, time
$\underline{t}$	modal disturbance vector
T	discretization interval
u	control element
$\underline{u}$	control vector
$\underline{\underline{U}}$	matrix defined in (3.17)
$\underline{v}$	left eigenvector
$\underline{\underline{V}}$	left eigenvector matrix
$\underline{w}$	right eigenvector
$\underline{\underline{W}}$	right eigenvector matrix
x	state variable
$\underline{x}$	state vector
y	output element
$\underline{y}$	output vector
z	modal state variable
$\underline{z}$	modal state vector

Greek

α	scalar defined in (2.40), objective function defined in (2.70)
β	scalar defined in (2.51), objective function defined in (2.71)
δ_{ij}	Kronecker delta
Δ	partition of a matrix
$\underline{\Delta}$	discrete control coefficient matrix
$\underline{\Theta}$	discrete disturbance coefficient matrix
$\underline{\Phi}$	discrete state coefficient matrix
λ	eigenvalue
$\underline{\Lambda}$	eigenvalue matrix
ν	index
ξ	modal activation, scalar defined in (2.52)
Π	product
$\sum$	sum
τ	time
ψ	degree of controllability

Superscript

$\cdot$	time derivative
$'$	perturbation variable
$*$	vector projection, arbitrary vector or matrix
T	matrix transpose
1	first partition of a matrix
2	second partition of a matrix
-1	matrix inversion

Subscript

-	vector
=	matrix
c	controllability
d	desired
i	element counter, run counter
j	element counter
ℓ	size of matrix partition
o	observability

Symbols

< >	dot product
><	dyadic product

Abbreviations

min	minimum
max	maximum
sign	signum

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